Chapter 6
Markets for Homogeneous Products

Only theory can separate the competitive from the anticompetitive.
—Robert Bork, The Antitrust Paradox

In this chapter we analyze the behavior of firms and consumer welfare under several oligopolistic market structures. The main assumption in this chapter is that the products are homogenous, meaning that consumers cannot differentiate among brands or distinguish among the producers when purchasing a specific product. More precisely, consumers cannot (or just do not bother) to read the label with the producer’s name on the product they buy. For example, non-brand-name products sold in most supermarkets—bulk fruit, vegetables, containers of grain—are generally purchased without having consumers learning the producer’s name.

In what follows, we assume that consumers are always price takers (henceforth, competitive) and have a well-defined aggregate-demand function. However, firms behave according to the assumed market structures analyzed below.

Our oligopoly analysis starts with section 6.1 (Cournot), which assumes that firms set their output levels simultaneously, believing that the output levels of their rival firms remain unchanged. Historically, as we discuss below, Cournot was the first to provide this modern treatment of oligopoly equilibrium. Section 6.2 (Sequential Moves) modifies the static Cournot setup, by assuming that firms move in sequence, and analyzes whether a firm benefits by setting its output level before any other one does. Following Bertrand’s criticism of the use of quantity produced as the actions chosen by firms, section 6.3 (Bertrand) analyzes
6.1 Cournot Market Structure


Cournot was central to the founding of modern mathematical economics. For the case of monopoly, the familiar condition where marginal-revenue equals marginal cost come directly from Cournot’s work (Shubik 1987). In chapter 7 of his book, Cournot employs the inverse-demand function to construct a system of firms’ marginal-revenue functions, which could be then solved for what we will call the Cournot output levels. Then, he introduced firms’ cost functions and the system of first-order conditions to be solved. Cournot did not consider the possibility that firms with sufficiently high cost may not be producing in this equilibrium.

In what follows, we develop the Cournot oligopoly model where firms sell identical products. In this model, firms are not price takers. Instead, each firm is fully aware that changing its output level will affect the market price.

6.1.1 Two-firm game

Let us consider a two-firm industry summarized by the cost function of each firm $i$ (producing $q_i$ units) given by

$$\mathcal{T}C_i(q_i) = c_iq_i, \ i = 1, 2, \ 	ext{where } c_2, c_1 \geq 0,$$  \hspace{1cm} (6.1)

and the market-demand function given by

$$p(Q) = a - bQ, \ a, b > 0, \ a > c_i, \ \text{where } Q = q_1 + q_2.$$  \hspace{1cm} (6.2)

In contrast to chapter 4, where we solved for a competitive equilibrium for this industry, here we solve for a Cournot oligopoly equilibrium. We first have to define a two-firm game that corresponds to a definition of a game given in Definition 2.1. Let each firm’s action be defined as choosing its production level, and assume that both firms choose their actions simultaneously. Thus, each firm $i$ chooses $q_i \in A_i \equiv [0, \infty)$, $i = 1, 2$. Also, let the payoff function of each firm $i$ be its profit function defined by $\pi_i(q_1, q_2) = p(q_1 + q_2)q_i - TC_i(q_i)$. Now, the game is properly defined since the players, their action sets, and their payoff functions are explicitly defined. All that is left to do now is to define the equilibrium concept.

**Definition 6.1** *The triplet* $(p^*, q_1^*, q_2^*)$ *is a Cournot-Nash equilibrium if*

1. *(a)* given $q_2 = q_2^*$; $q_1^*$ solves $\max_{q_1} \pi_1(q_1, q_2^*) = p(q_1 + q_2^*)q_1 - TC_1(q_1) = [a - b(q_1 + q_2^*)]q_1 - c_1q_1$

   *(b)* given $q_1 = q_1^*$; $q_2^*$ solves $\max_{q_2} \pi_2(q_1^*, q_2) = p(q_1^* + q_2)q_2 - TC_2(q_2) = [a - b(q_1^* + q_2)]q_2 - c_2q_2$

2. $p^* = a - b(q_1^* + q_2^*)$, $p^*, q_1^*, q_2^* \geq 0$.

That is, according to Definition 6.1, a Cournot equilibrium is a list of output levels produced by each firm and the resulting market price so that no firm could increase its profit by changing its output level, given that other firms produced the Cournot output levels. Thus, Cournot equilibrium output levels constitute a Nash equilibrium in a game where firms choose output levels.

Now that the equilibrium concept is well defined, we are left to calculate the Cournot equilibrium for this industry. Firm 1’s profit-maximization problem yields the first-order condition given by

$$0 = \frac{\partial \pi_1(q_1, q_2^*)}{\partial q_1} = a - 2bq_1 - bq_2 - c_1$$

which yields the familiar profit-maximizing condition in which each firm (firm 1 in this equation) sets its marginal revenue ($MR(q_1) = a - 2bq_1 - bq_2$) equal to marginal cost ($c_1$). The second-order condition guaranteeing a global maximum is satisfied since $\frac{\partial^2 \pi_1}{\partial q_1^2} = -2b < 0$ for every $q_1$ and $q_2$. Solving for $q_1$ as a function of $q_2$ yields the best-response...
Markets for Homogeneous Products

function (also commonly known as reaction function) of firm 1, which we denote by \( R_1(q_2) \). Hence,

\[
q_1 = R_1(q_2) = \frac{a - c_1}{2b} - \frac{1}{2} q_2.
\]  

(6.3)

Similarly, we can guess that firm 2's best-response function is given by

\[
q_2 = R_2(q_1) = \frac{a - c_2}{2b} - \frac{1}{2} q_1.
\]  

(6.4)

The best-response functions of the two firms are drawn in Figure 6.1 in the \((q_1, q_2)\) space.

![Graph showing best-response functions](image)

Figure 6.1: Cournot best-response functions (the case for \( c_2 > c_1 \))

The two best-response functions are downward sloping, implying that for each firm, if the rival's output level increases, the firm would lower its output level. The intuition is that if one firm raises its output level, the price would drop, and hence in order to maintain a high price the other firm would find it profitable to decrease its output level. A perhaps more intuitive explanation for why a firm's best-response function is downward sloping is that an increase in a rival's output shifts the residual demand facing a firm inward. Hence, when a firm faces a lower demand it would produce a smaller amount.

Now, the Cournot equilibrium output levels can be calculated by solving the two best-response functions (6.3) and (6.4), which correspond to the intersection of the curves illustrated in Figure 6.1. Thus,

\[
q_1^* = \frac{a - 2c_1 + c_2}{3b} \quad \text{and} \quad q_2^* = \frac{a - 2c_2 + c_1}{3b}.
\]  

(6.5)

6.1 Cournot Market Structure

Hence, the aggregate industry-output level is \( Q^* = q_1^* + q_2^* = \frac{2a - c_1 - c_2}{3b} \), and the Cournot equilibrium price is

\[
p^* = a - bQ^* = \frac{a + c_1 + c_2}{3}.
\]  

(6.6)

It is easy to confirm from (6.5) that the output of the high-cost firm is lower than the output level of the low-cost firm. That is, \( c_2 \geq c_1 \) implies that \( q_1 \geq q_2 \).

Altogether, the Cournot profit (payoff) level of firm \( i \), as a function of the unit costs for firms \( i \) and \( j \), \( i \neq j \), is given by

\[
\pi_i^* = (p^* - c_i)(q_i^*) = \left( \frac{a + c_1 + c_j}{3} - c_i \right) \left( \frac{a - 2c_1 + c_j}{3b} \right)
\]

\[
= \frac{(a - 2c_1 + c_j)^2}{9b} = b q_i^* - b q_i^*.
\]  

(6.7)

We conclude this section with some comparative static analysis. Suppose that firm 1 invents a new production process that reduces its unit production cost from \( c_1 \) to \( \bar{c}_1 \), where \( \bar{c}_1 < c_1 \). The type of R&D leading to cost reduction is called "process innovation," to which we will return in Chapter 9. Equation (6.5) implies that \( q_1^* \) increases while \( q_2^* \) decreases. This is also shown in Figure 6.1, where a decrease in \( c_1 \) shifts \( R_1(q_2) \) to the right, thereby increasing the equilibrium \( q_1^* \) while decreasing \( q_2^* \). Also, (6.6) implies that a decrease in \( c_1 \) (or \( c_2 \)) would decrease the equilibrium price \( p^* \), and (6.7) implies that a decrease in \( c_1 \) would increase the profit of firm 1 while lowering the profit of firm 2.

6.1.2 \( N \)-seller game

Suppose now the industry consists of \( N \) firms, \( N \geq 1 \). We analyze two types of such industries: (a) \( N \) identical firms, all having the same cost function, or (b) heterogeneous firms, where some firms have cost functions different from others. Since solving the general case of firms with different cost functions would require solving \( N \) first-order conditions (intersecting \( N \) best-response functions), we first solve the model by assuming that all firms have identical technologies. That is, \( c_i = c \) for every \( i = 1, 2, \ldots, N \). In the appendix (section 6.7) we introduce a procedure that makes solving the heterogeneous-firms case easy.

Since all firms have the same cost structure, the first step would be to pick up one firm and calculate its output level as a function of the output levels of all other firms. In other words, we would like to calculate the best-response function of a representative firm. With no loss of generality, we derive the best-response function of firm 1. Thus,
6.1 Cournot Market Structure

Now, we let the number of firms grow with no bounds, \((N \to \infty)\). Then, we have it that

\[
\lim_{N \to \infty} q^f = 0, \quad \text{and} \quad \lim_{N \to \infty} Q^f = \lim_{N \to \infty} \left( \frac{a - c}{b} \right) \left( \frac{N}{N + 1} \right) = \left( \frac{a - c}{b} \right).
\]

That is, in a Cournot equilibrium, as the number of firms grows indefinitely, the output level of each firm approaches zero whereas the industry's aggregate output level approaches the competitive output level given in Proposition 4.1. Also,

\[
\lim_{N \to \infty} p^f = \lim_{N \to \infty} \frac{a}{N + 1} + \frac{c}{N + 1} = c = p^c.
\]

Hence, the Cournot equilibrium price approaches the competitive price that equals the unit production cost of a firm (see Proposition 4.1). These results often cause some confusion among students, leading them to believe that competitive behavior occurs only when there are many (or infinitely many) firms. However, as we pointed out in chapter 4, we can assume a competitive market structure for any given number of firms, and even solve for a competitive equilibrium for the case where \(N = 1\). What equations (6.11) and (6.12) say is that the Cournot market structure yields approximately the same price and industry output as the competitive market structure when the number of firms is large.

6.1.3 Cournot equilibrium and welfare

Since our analysis starts with given demand functions (rather than the consumers' utility functions), we cannot measure the social welfare by calculating consumers' equilibrium-utility levels. Instead, we approximate social welfare by adding consumers' surplus and firms' profits (see subsection 3.2.3 on page 52 for a justification of this procedure of welfare approximation). Note that profit should be part of the economy's welfare because the firms are owned by the consumers, who collect the profits via firms' distributions of dividends.

Substituting the Cournot equilibrium price (6.10) into (3.3) on page 52, we obtain the consumers' surplus as a function of the number of firms, \(N\). Hence, \(CS^c(N) = \frac{N^2(a - c)^2}{2(N + 1)^2}\). Clearly, \(\frac{dCS^c(N)}{dN} > 0\), meaning that consumers' surplus rises with the entry of more firms, due to the reduction in price and the increase in the quantity consumed.

We define social welfare as the sum of consumers' surplus plus the industry aggregate profit (see section 4.3 on page 68 for a definition). Thus, if we recall (6.10),

\[
W^c(N) = CS^c(N) + N\pi^c(N)
\]
6.2 Sequential Moves

In the previous section, we analyzed industries where firms strategically choose their output levels. All those games were static in the sense that players simultaneously choose their quantity produced. In this section, we assume that the firms move in sequence. For example, in a two-firm, sequential-moves game, firm 1 will choose its output level before firm 2 does. Then, firm 2, after observing the output level chosen by firm 1, will choose its output level, and only then will output be sold and profits collected by the two firms. This type of market structure is often referred to as Leader-Follower on the basis of von Stackelberg’s work (1934) (see Konow 1994 von Stackelberg’s biography). This type of behavior defines an extensive form game studied in section 2.2.

In this section we do not raise the important question of what determines the order of moves, that is, why one firm gets to choose its output level before another. We return to this question in chapter 8, where we distinguish among established firms (called incumbent firms) and potential entrants. Here, we assume that the order of moves is given, and we develop the tools for solving an industry equilibrium under a predetermined order of moves.

We analyze a two-stage game, where firm 1 (the leader) chooses the quantity produced in the first stage. The quantity chosen in the first stage is irreversible and cannot be adjusted in the second stage. In the second stage, only firm 2 (the follower) chooses how much to produce after observing the output level chosen by firm 1 in the first stage. Here, the game ends after the second stage, and each firm collects its profit. Our main questions are (a) Is there any advantage for moving in the first stage rather than the second? and (b) How would the equilibrium market price and production levels compare to the static Cournot equilibrium price and output levels?

Following Definition 2.9 on page 26, this game has a continuum of subgames indexed by the output level chosen by firm 1 in the first stage. A finite-horizon dynamic game is generally solved backwards. We look for a subgame perfect equilibrium (Definition 2.10 on page 27) for this game. Hence, we first analyze the players’ (firm 2 in our case) action in

Also, note that \( \frac{8W(N)}{2N} > 0 \). Hence, although the industry profit declines with an increase in the number of firms, the increase in consumers’ surplus dominates the reduction in the industry profit. Thus, in this economy, free entry is welfare improving!

6.2 Sequential Moves

the last period, assuming that the actions played in previous period are given. Then, we go one period backwards, and analyze firm 1’s action given the strategy (see Definition 2.8 on page 24) of how firm 2 chooses its output level based on the first-period action. To simplify the exposition, let all firms have identical unit cost, \( c_1 = c_2 = c \).

The second-period subgames

In the second period, only firm 2 moves and chooses \( q_2 \) to maximize its profit, taking firm 1’s quantity produced, \( q_1 \), as given. As you probably noticed, we have already solved this problem before, since the second-period problem of firm 2 is identical to the problem firm 2 solves in a Cournot market structure. This maximization results in the best-response function of firm 2 given in (6.4). Hence, \( R_2(q_1) = \frac{a - c}{2b} - \frac{1}{2} q_1 \). Note that the function \( R_2(q_1) \) constitutes firm 2’s strategy for this game, since it specifies its action for every possible action chosen by firm 1.

The first-period game

In period 1, firm 1 calculates \( R_2(q_1) \) in the same way as firm 2. Thus, firm 1 is able to calculate how firm 2 will best reply to its choice of output level. Knowing that, firm 1 chooses \( q_1^* \) to

\[
\max_{q_1} p(q_1 + R_2(q_1))q_1 - c_1 = \left[ a - b \left( q_1 + \frac{a - c}{2b} - \frac{q_1}{2} \right) \right] q_1 - c_1.
\]

We leave it to the reader to derive the first- and second-order conditions. Thus, the quantity produced by the leader is

\[
q_1^* = \frac{a - c}{2b} = \frac{3}{2} q_1^* > q_1^*.
\]

Hence, under the sequential-moves market structure, the leader produces a higher level of output than the Cournot market structure. Substituting (6.15) into \( R_2(q_1) \) yields the followers’ equilibrium-output level:

\[
q_2^* = \frac{a - c}{4b} = \frac{3}{4} q_2^* < q_2^*
\]

implying that the follower’s output level falls compared with the Cournot output level. Thus, the leader’s gain in output expansion comes partly from the reduction in the follower’s output level. The equilibrium price and aggregate output levels are given by

\[
p^* = \frac{a + 3c}{4} < \frac{a + 2c}{3} = p^c \quad \text{and} \quad Q^* = \frac{3(a - c)}{4b} > 2 \frac{a - c}{3b} = Q^c.
\]

Therefore,
Proposition 6.1 A sequential-moves quantity game yields a higher aggregate industry-output level and a lower market price than the static Cournot market structure.

Thus, the equilibrium market outcome under a sequential-moves game is more competitive than the Cournot equilibrium outcome in the sense that this outcome is somewhere in between the competitive equilibrium outcome derived in chapter 4 and the Cournot outcome derived in section 6.1. The intuition behind Proposition 6.1 is as follows: Under the Cournot market structure, firm 1 perceives the output produced by firm 2 as given. However, under sequential-moves market structure, firm 1 knows firm 2's best-response function and therefore calculates that firm 2 will reduce its output level in response to its increase in output level. Hence, when firm 1 expands output, it expects the price to fall faster under Cournot than under sequential-moves market structure. Therefore, in order maintain a high price, firm 1 will produce more under the sequential game than it will under Cournot. Now, (6.15) and (6.16) demonstrate that the increase in aggregate output stems from the fact that the follower does not find it profitable to cut its output level by the same amount as the increase in the leader's output level. This happens because the reaction functions are sloped relatively flat (slope is negative but exceeds -1), implying that a firm reduces its output level by less than the increase in the output level of the rival firm.

We now compare firms' profit levels under sequential moves to the Cournot profit levels. We leave it to the reader to verify that the leader's profit increases while the follower's declines. That is,

\[ \pi_1^* = \frac{(a-c)^2}{8b} > \pi_1^c \quad \text{and} \quad \pi_2^* = \frac{(a-c)^2}{16b} < \pi_2^c, \]  

where \( \pi_1^c \) and \( \pi_2^c \) are given in (6.7). Note that we could have concluded even without going into the precise calculations that the leader's profit under the sequential-game equilibrium will be higher than under the Cournot. How? It is very simple! Since firm 2 reacts in a "Nash fashion," firm 1 could just choose to produce the Cournot output level \( q^c \). In this case, firm 1 would earn exactly the Cournot profit. However, since in the sequential game firm 1 chooses to produce a different output level, it must be increasing its profit compared with the Cournot profit level. The kind of reasoning we just described is called a revealed profitability argument, and the reader is urged to learn to use this kind of reasoning whenever possible because performing calculations to investigate economic effects does not generate an intuitive explanation for these effects. In contrast, logical deduction often provides the necessary intuition for understanding economic phenomena.

6.3 Bertrand Market Structure

Finally, we can logically deduce how industry profit under sequential moves compare with industry profit under Cournot. Equations (6.17) show that the market price under sequential moves is lower than it is under Cournot. Since the Cournot market price is lower than the monopoly's price, and since monopoly makes the highest possible profit, it is clear that industry profit must drop when we further reduce the price below the monopoly's price. Hence, whenever \( c_1 = c_2 \), industry profit must be lower under sequential moves. In a more general environment, this argument may not holds when the industry profit is not a concave function of \( p \).

6.3 Bertrand Market Structure

In a Cournot market structure firms were assumed to choose their output levels, where the market price adjusted to clear the market and was found by substituting the quantity produced into consumers' demand function. In contrast, in a Bertrand market structure firms set prices rather than output levels. The attractive feature of the Bertrand setup, compared with the Cournot market structure, stems from the fact that firms are able to change prices faster and at less cost than to set quantities, because changing quantities will require an adjustment of inventories, which may necessitate a change in firms' capacity to produce. Thus, in the short run, quantity changes may not be feasible, or may be too costly to the seller. However, changing prices is a relatively low-cost action that may require only a change in the labels displayed on the shelves in the store. Let us turn to the Bertrand market structure.

In 1838 Joseph Bertrand published a review of Cournot's book (1838) harshly critical of Cournot's modeling. It seems, however, that Bertrand was dissatisfied with the general modeling of oligopoly rather than with the specific model derived by Cournot. Today, most economists believe that quantity and price oligopoly games are both needed to understand a variety of markets. That is, for some markets, an assumption that firms set quantities may yield the observed market price and quantity produced, whereas for others, a price-setting game may yield the observed market outcomes. Our job as economists would then be to decide which market structure yields a better approximation of the observed price and quantity sold in each specific market.

We now analyze the two-firm industry defined in (6.1) and (6.2) and look for a Nash equilibrium (see Definition 2.4) in a game where the two firms use their prices as their actions. First, note that so far, our analysis has concentrated on a single market price determined by our assumption that consumers are always on their demand curve. However, in a Bertrand game we have to consider outcomes where each firm
6.3 Bertrand Market Structure

6.3.1 Solving for Bertrand equilibrium

Before we characterize the Bertrand equilibria, it is important to understand the discontinuity feature of this game. In the Cournot game, the payoff (profit) functions are continuous with respect to the strategic variables (quantities); in the Bertrand price game, by contrast, equation (6.19) exhibits a discontinuity of the payoff functions at all the outcomes where \( p_1 = p_2 \). That is, if one firm sells at a price that is one cent higher than the other firm, it would have a zero market share. However, a two-cent price reduction by this firm would give this firm a one 100 percent market share. The action of a firm to slightly reduce the price below that of its competitor is called undercutting. Since undercutting involves setting a price slightly lower than the competitor's, we need to examine the types of currencies used in order to determine the smallest possible undercutting actions available to firms. Therefore, we make the following definition:

\[ \text{Definition 6.3} \text{ Let } \epsilon \text{ be the smallest possible monetary denomination (smallest legal tender). The medium of exchange (money) is said to be continuous if } \epsilon = 0, \text{ and discrete if } \epsilon > 0. \]

Examples of discrete smallest legal tenders are: in China, \( \epsilon = 1 \text{ Fen} \); in Israel, \( \epsilon = 5 \text{ Agorot} \); and in the US, \( \epsilon = 1 \text{ cent} \).

The following proposition characterizes Bertrand equilibria.

\[ \text{Proposition 6.2} \]

1. If the medium of exchange is continuous and if the firms have the same cost structure, \( (c_2 = c_1 = c) \), then a Bertrand equilibrium is \( p_1^* = p_2^* = c \), and \( q_1^* = q_2^* = (a - c)/(2b) \).

2. Let the medium of exchange be discrete, and assume that \( c_2 \) is denominated in the medium of exchange. That is, \( c_2 = \lambda \epsilon \), where \( \lambda \geq 1 \) is an integer. Also let \( \epsilon \) be sufficiently small, that is, satisfying \( (c_2 - \epsilon - c_1) \left( \frac{a(c_2 - c_1)}{b} \right) > (c_2 - c_1) \left( \frac{a + c_2}{2b} \right) \). Then, if \( c_2 - c_1 > \epsilon \), the unique Bertrand equilibrium is \( p_2 = c_2, p_1 = c_2 - \epsilon, q_2^* = 0, \) and \( q_1^* = (a - c_2 + \epsilon)/b \).

Thus, if firms have equal unit costs, the Bertrand equilibrium price and aggregate output are the same as for the competitive equilibrium. In other words, undercutting reduces the prices to marginal cost. In cases where firm 1 has a lower unit cost than firm 2, firm 1 undercut firm 2 by charging the highest possible price that is lower than \( c_2 \), which is given by \( p_1 = c_2 - \epsilon \).
Proof. Part 1: In equilibrium, each firm must make nonnegative profit. Hence, \( p_i^* \geq c_i, \ i = 1, 2 \).

We first establish that in a Bertrand equilibrium both firms charge the same prices. By way of contradiction suppose that \( p_1^* > p_2^* > c \). Then, by (6.19), firm 1 makes zero profit. However, since the medium of exchange is continuous, firm 1 can increase its profit by reducing its price to \( p_1 = p_1^* - \epsilon \), where \( \epsilon \) can be as small as one wants, thereby grabbing the entire market, thereby making strictly positive profit, a contradiction.

By way of contradiction suppose that \( p_1^* = p_2^* = c \). Then, since the medium of exchange in continuous, firm 2 can raise its price slightly while still maintaining a lower price than firm 1. Hence, firm 2 will deviate, a contradiction.

Now that we have established that \( p_1^* = p_2^* \), by way of contradiction assume that \( p_1^* = p_2^* > c \). Clearly, this cannot constitute a Nash equilibrium in prices since firm 1, say, would have an incentive unilaterally to reduce its price to \( p_1 = p_1^* - \epsilon \), where \( \epsilon \) can be as small as one wants, thereby grabbing the entire market. For \( \epsilon \) sufficiently small, this deviation is profitable for firm 1.

Part 2: To briefly sketch the proof of part 2, observe that firm 2 makes a zero profit and cannot increase its profit by unilaterally raising its price above \( p_2^* = c_2 \). Hence, firm 2 does not deviate. Now, for firm 1 to be able to sell a positive amount, it must set \( p_1^* \leq c_2 \). If \( p_1^* = c_2 = p_2^* \), then (6.19) implies that the firms split the market by selling each \( q_i = a/(2b) \). In this case, the profit of firm 1 is

\[
\pi_1 = (c_2 - c_1) q_1 = (c_2 - c_1) \frac{a - c_2}{2b}.
\]

(6.20)

However, if firm 1 undercuts the smallest legal tender, then it becomes the sole seller and sells \( q_1 = \frac{a - (c_2 - c)}{b} \). In this case,

\[
\pi_1 = (c_2 - \epsilon - c_1) q_1 = (c_2 - \epsilon - c_1) \frac{a - (c_2 - \epsilon)}{b}.
\]

(6.21)

Comparing (6.20) with (6.21) yields the condition stated in part 2.

6.3.2 Bertrand under capacity constraints

The previous section demonstrated that when the firms have the same cost structure, price competition reduces prices to unit costs, thereby making firms earn zero profits. Economists often feel uncomfortable with this result, especially since it makes the number of firms in the industry irrelevant, in the sense that under symmetric Bertrand competition, price drops to unit cost even when there are only two firms. Now, if most industries are indeed engaged in a Bertrand competition as described in this section, then we should observe unit-cost prices for those industries with two or more firms. If this case is realistic, then the antitrust authority should not have to worry about industries' concentration levels and should devote all its effort to fighting monopolies. Clearly, we rarely observe intense price competition among industries with a small number of firms, and therefore the antitrust authority challenges mergers of firms that lead to highly concentrated industries (see Section 8.6).

One way to overcome this problem is to follow Edgeworth (1925) and to assume that in the short run, firms are constrained by given capacity that limits their production levels. The British economist Francis Ysidro Edgeworth, who made enormous contributions to economic theory and other disciplines, identified some discontinuity properties of the firms' profit functions when firms produce under increasing marginal cost (decreasing returns to scale) technologies. In Edgeworth's words (Edgeworth 1925, 118):

"In the last case there will be an intermediate tract through which the index of value will oscillate, or rather vibrate irregularly for an indefinite length of time. There will never be reached that determinate position of equilibrium which is characteristic of perfect competition."

We demonstrate Edgeworth's argument by assuming an extreme version of increasing marginal cost, which is letting the cost of expanding production beyond a certain output level (which we call capacity) be infinite. Figure 6.2 illustrates a market-demand curve composed of four consumers, each buying, at most, one unit.

Figure 6.2 assumes that consumer 1 is willing to pay a maximum of $3 for one unit, consumer 2 a maximum of $2, consumer 3 a maximum of $2, and consumer 4 will not pay at all. Such prices are commonly termed as consumers' reservation prices.

Suppose now that there are two firms and that each is capable of producing at zero cost, \( c_1 = c_2 = 0 \). Then, Proposition 6.2, proved in the previous subsection, shows that if firms are not subject to capacity constraints, then Bertrand competition would lead to prices of zero, \( p_1^* = p_2^* = 0 \).

To demonstrate Edgeworth's argument, suppose now that in the short run each firm is limited to producing, at most, one unit. Then, it is easy to show that the prices \( p_1 = p_2 = 0 \) no longer constitute a Nash equilibrium. To see this, observe that firm 1 can increase its profit from \( \pi_1 = 0 \) to \( \pi_1 = 3 \) by increasing its price to \( p_1 = 3 \), and selling its unit to the consumer with the highest reservation price. In this outcome, firm 1 sells one unit to the consumer with a reservation price of 3,
6.4 Cournot versus Bertrand

In sections 6.1 and 6.3 we analyzed the same industry where in the Cournot-market-structure firms use quantity produced as actions, whereas in the Bertrand-market-structure firms use prices as actions. The analyses of these sections show that in general, the two types of market structures yield different market outcomes (prices and quantity produced). Thus, when we change the firms' actions from choosing quantities to choosing prices, the Nash equilibrium yields a completely different outcome because under Cournot, firms make positive profit, since the resulting market price exceeds unit cost, whereas under Bertrand, prices drop to unit cost. Moreover, in a Bertrand game, only the low-cost firm produces, which is generally not the case for the Cournot game. Therefore, we can state that in a one-shot (static) game there is no correspondence between the Cournot solution and the Bertrand solution.

However, Kreps and Scheinkman (1983) constructed a particular environment (a particular two-period dynamic game) where, in the first period, firms choose quantity produced (accumulate inventories) and in the second period, the quantities are fixed (cannot be changed) and firms choose prices. They showed that the quantities chosen by firms in the first period and the price chosen in the second period are exactly the Cournot outcome given in (6.5) and (6.6). That is, they show that for some market games where two firms choose how much to produce in period 1, and then set prices in period 2, a subgame perfect equilibrium (see Definition 2.10 on page 27) yields the exact quantity produced and price as those in a one-shot Cournot-market-structure game, where firms choose only how much to produce.

We will not bring a complete proof of their proposition; however, we illustrate the idea in our simple two-firm industry for the case where \( p = 10 - Q \), and both firms have a unit cost of \( c = 1 \).

As we discussed earlier, the easiest way of solving for a subgame perfect equilibrium for a dynamic finite game is to solve it backwards. Therefore, we begin with the second period and ask what prices will be chosen by firms in a Nash-equilibrium one-shot price game, where the quantity produced is taken as given by first-period choices. Then, we analyze the first period looking for a subgame perfect equilibrium in first-period production levels, where firms can calculate and take into account the second-period equilibrium market prices, which depend on first-period production levels.

The second-period subgame

Assume that for some reason, the firms choose to produce the Cournot capacity levels \( q_1^* = q_2^* = 3 \). Hence, total industry output is \( Q^c = 6 \). We now show that in a Nash equilibrium for the second-period subgame both firms will choose to set prices that clear the market under the Cournot outcome. That is, each firm will set \( p_i = 4 = p^c \). Figure 6.3
illustrates the Cournot outcome.

![Diagram](image)

Figure 6.3: Residual demand when firms have fixed inventories

Note that in the second period, firms are free to choose any price they wish so that the Nash equilibrium prices may differ from \( p^e = 4 \). To demonstrate that this is not the case, we now show that given \( p^e = 4 \), firm 1 will not deviate and will also choose \( p_1 = 4 \). First, note that firm 1 will not lower its price below \( p_1 = 4 \) because a price reduction will not be followed by an increase in sales (the capacity is limited to \( q_1 = 3 \)). Thus, lowering the price will only lower its revenue.

Second, we must show that firm 1 cannot increase its profit by raising its price and selling less than \( q_1^* = 3 \). The right side of Figure 6.3 exhibits the residual demand facing firm 1 when it raises its price above \( p_1^* = 4 \). Residual demand is the demand facing firm 1 after the quantity supplied by firm 2 is subtracted from the aggregate industry demand. In the present case, we subtract \( q_2^* = 3 \) from the aggregate demand curve to obtain the residual demand curve facing firm 1, given by \( q_1 = 10 - p - 3 = 7 - p \) or its inverse \( p = 7 - q_1 \). The most important observation to be made about Figure 6.3 is that the marginal-revenue curve derived from this residual-demand function \( (MR_1(q_1) = 7 - 2q_1) \) is strictly positive for all output levels satisfying \( q_1 \leq 3.5 \), implying that the residual demand is elastic at this interval. Therefore, increasing \( p_1 \) will only reduce the revenue of firm 1. This establishes the following claim.

\[
\text{Lemma 6.1} \quad \text{If the output (capacity) levels chosen in period 1 satisfy} \quad q_1 + q_2 \leq 6, \text{ then the Nash equilibrium exhibits both firms choosing the market-clearing price in the second period.}
\]

Lemma 6.1 shows that, given firms' choices of output levels, in the second-period price game firms will strategically choose to play the market price that clears the market at the given aggregate output level.

### 6.5 Self-Enforcing Collusion

The first-period game

In the first period, firms observe that the second-period price would be the market-clearing price (Lemma 6.1). Therefore, for each firm, the first-period-capacity-choice problem is precisely the Cournot-quantity-choice problem as formulated in Definition 2.4. Hence, in the first period, firms would choose the Cournot quantity levels \( p_1^* = p_2^* = 3 \). Intuitively, in the first period both firms know that the second-period price choices by both firms would be the price that clears the market for the first-period production levels. This knowledge makes the firms' first-period-output-choice problem identical to firms' output choices in a Cournot market structure as defined in Definition 6.1.

Finally, note that this illustration does not provide a complete proof for this statement, since in Lemma 6.1 we assumed that the firms did not choose "very high" capacity levels in the first period. In that respect, Lemma 6.1 is not proven for output levels exceeding \( q_1 + q_2 > 6 \). We refrain from proving that in order to avoid using mixed strategies in this book. Also, from time to time this result causes some confusion among students and researchers, leading them to state that there is no reason for using Bertrand price competition anymore since the two-period, capacity-price game would yield the same outcome as the Cournot market structure. Note that this statement is too strong, since it holds only for the particular two-period game analyzed in the present section.

### 6.5 Self-Enforcing Collusion

In this section we extend the basic static Cournot game to an infinitely repeated game in which firms produce output and collect profits in each period. Although the analysis in this section is self-contained, the reader is urged to some background on repeated games by reading section 2.3.

One very important result will emerge from analyzing an infinitely repeated Cournot game, namely, that the outcome in which all firms produce the collusive output levels (see the cartel analysis in subsection 5.4.1) constitutes a subgame perfect equilibrium for the noncooperative repeated Cournot game. More precisely, in subsection 6.1.2 we proved that under the Cournot market structure with two or more firms, aggregate industry output exceeds the monopoly output level (which equals the cartel's total output level). Moreover, we showed that as the number of firms increases, the output level increases and converges to the competitive output level. Altogether, firms have a lot to gain by colluding rather than competing under any market structure. In this section we show that if the Cournot game is repeated infinitely, then
the collusive output level can emerge as a noncooperative equilibrium. The importance of this result is that it implies that observing an industry where production levels are limited and firms make strictly positive profits does not imply that the firms are engaged in any cooperative activities. In fact, what we show in this section is that the cooperative collusive output levels can be sustained as a noncooperative equilibrium.

In the subsection 6.5.1 we develop a simple Cournot duopoly model and analyze the incentives to collude among firms and the incentive for each firm to unilaterally deviate from collusion when the game is played only once. Subsection 6.5.2 analyzes equilibrium outcome when the one-shot game is repeated infinitely.

6.5.1 The one-shot game

Consider the following basic one-shot Cournot game: There are two firms denoted by \( i = 1, 2 \). We denote by \( q_i \) the output level of firm \( i \). The demand facing the industry is \( p = 1 - q_1 - q_2 \). Let \( Q \equiv q_1 + q_2 \) denote the aggregate industry-output level, and assume that production is costless.

In the following subsections we quickly derive the already familiar Cournot duopoly equilibrium, the collusion (cooperative) monopoly equilibrium, and then the incentives to deviate from the cooperative outcome.

**Duopoly: Non-cooperative behavior**

In view of Definition 6.1, in a Cournot market structure firm 1 maximizes \( \pi_1 = (1 - q_1 - q_2)q_1 \), yielding a best-response function: \( q_1(q_2) = (1 - q_2)/2 \) and the equilibrium output levels \( q_1 = q_2 = 1/3 \equiv M \), where \( M \) stands for medium production level. Hence, \( Q = 2/3 \), and \( p = 1/3 \), implying that \( \pi_i = 1/9 \). The profits of the firms under duopoly are displayed in the second column and second row of Table 6.1.

**Table 6.1:** Cooperation \( L \); Noncooperative Cournot duopoly \( M \); Defection from cooperation \( H \).

<table>
<thead>
<tr>
<th>Firm 2</th>
<th>Firm 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_2 = L = 1/4 )</td>
<td>( q_1 = L = 1/4 )</td>
</tr>
<tr>
<td>( q_2 = M = 1/3 )</td>
<td>( q_1 = M = 1/3 )</td>
</tr>
<tr>
<td>( q_2 = H = 3/8 )</td>
<td>( q_1 = H = 3/8 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( q_1 )</th>
<th>( q_2 )</th>
<th>( q_1 )</th>
<th>( q_2 )</th>
<th>( q_1 )</th>
<th>( q_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/8</td>
<td>5/8</td>
<td>5/8</td>
<td>9/8</td>
<td>36</td>
<td>36</td>
</tr>
<tr>
<td>1/8</td>
<td>1/8</td>
<td>5/8</td>
<td>5/8</td>
<td>36</td>
<td>36</td>
</tr>
<tr>
<td>1/8</td>
<td>1/8</td>
<td>1/8</td>
<td>7/8</td>
<td>48</td>
<td>48</td>
</tr>
<tr>
<td>5/8</td>
<td>5/8</td>
<td>1/8</td>
<td>7/8</td>
<td>48</td>
<td>48</td>
</tr>
<tr>
<td>5/8</td>
<td>5/8</td>
<td>1/8</td>
<td>7/8</td>
<td>48</td>
<td>48</td>
</tr>
<tr>
<td>5/8</td>
<td>5/8</td>
<td>1/8</td>
<td>7/8</td>
<td>48</td>
<td>48</td>
</tr>
<tr>
<td>5/8</td>
<td>5/8</td>
<td>1/8</td>
<td>7/8</td>
<td>48</td>
<td>48</td>
</tr>
</tbody>
</table>

6.5.2 The infinitely repeated game

Suppose now that the two firms live forever. The game proceeds as follows: In each period \( t \) both firms observe what both firms played in

**Collusion:** Cooperative behavior

We assume that when the two firms collude, they act as a cartel, analyzed in subsection 5.4.1. Since the firms have identical technologies that exhibit constant returns to scale, the present case is easy to analyze because under CRS there is no difference whether under collusion they operate one or two plants. In any case, the cartel’s profit-maximizing output is found by equating \( MR(Q) = 1 - 2Q = 0 = MC_i \), implying that \( Q = 1/2, p = 1/2 \). Hence, equal division of output between the two colluding firms imply that \( q_i = L = 1/4 \), where \( L \) stands for “low” output levels. Thus, as expected, collusion implies that both firms restrict their output levels below the Cournot output levels. The two firms equally divide the profit, so \( \pi_i = pQ/2 = 3/8 \), which is displayed in the first column and row in Table 6.1.

**Deviation from collusion**

Suppose that firm 2 plays the naive collusive output level \( q_2 = L \). We now show that in this one-shot game, firm 1 can increase its profit by unilaterally increasing its output level. To see that, for given \( q_2 = 1/4 \), firm 1 chooses \( q_1 \) to maximize \( \pi_1 = (1 - q_1 - 1/4)q_1 \), yielding \( 0 = 3/4 - 2q_1 \). Hence, \( q_1 = 3/8 \equiv H \). Thus, if firm 2 does not deviate from \( q_2 = L \), firm 1 has the incentive to increase its output to a high level. In this case, \( Q = 3/8 + 1/4 = 5/8 \), \( p = 3/8 \), \( \pi_1 = 9/64 \) and \( \pi_2 = 3/32 \); both are displayed in the first column, third row in Table 6.1.

**Equilibrium in the one-shot game**

The first part of the next proposition follows directly from equation (6.5) and also from Table 6.1. The second part follows from Definition 2.6 and Table 6.1.

**Proposition 6.3** In the one-shot game:

1. there exists a unique Cournot-Nash equilibrium, given by \( q_1 = q_2 = M = 1/3 \);
2. the equilibrium outcome is Pareto dominated by the “cooperative outcome” \( q_1 = q_2 = L = 1/4 \).

Note that we use the Pareto criterion to refer only to the profit of firms, thereby disregarding consumers’ welfare.
all earlier periods (observe period \( t \) history as defined in Definition 2.11) and play the one-shot game described in Table 6.1. That is, in each period \( t \), each firm \( i \) chooses \( q_i(t) \), where \( q_i(t) \in \{L, M, H\} \), \( i = 1, 2 \) and \( t = 0, 1, 2, \ldots \). A strategy of firm \( i \) is a list of output levels chosen each period by firm \( i \) after the firm observed all the output levels chosen by each firm in all earlier periods (see Definition 2.11 for a precise definition of a strategy in repeated games).

Let \( 0 < \rho < 1 \) be the discount factor. Note that in perfect capital markets, the discount factor is inversely related to the interest rate. Let \( r \) denote the interest rate. Then, \( \rho = \frac{1}{1+r} \). As \( r \) rises, \( \rho \) falls, meaning that future profits are less valuable today. Following Assumption 2.1, we assume that the objective of each firm is to maximize the sum of present and discounted future profits given by

\[
\Pi_i = \sum_{t=1}^{\infty} \rho^{t-1} \pi_i(t)
\]  

(6.22)

where the values of \( \pi_i(t) \) are given in Table 6.1.

The trigger strategy

We restrict the discussion here to one type of strategies called trigger strategies, meaning that in every period \( \tau \) each player cooperates (playing \( q_i(\tau) = L \)) as long as all players (including himself) cooperated in all periods \( t = 1, \ldots, \tau - 1 \) (see Definition 2.11 for a precise definition). However, if any player deviated in some period \( \tau \in \{0, 1, 2, \ldots, \tau - 1\} \), then player \( i \) plays the noncooperative (duopoly) strategy forever! That is, \( q_i(t) = M \) for every \( t = \tau, \tau + 1, \tau + 2, \ldots \). Formally, let us restate Definition 2.12 for the present game.

**Definition 6.4** Player \( i \) is said to be playing a trigger strategy if for every period \( \tau = 1, 2, \ldots, \)

\[ q_i(\tau) = \begin{cases} L & \text{as long as } q_i(t) = q_2(t) = L \text{ for all } t = 1, \ldots, \tau - 1 \\ M & \text{Otherwise.} \end{cases} \]

In other words, firm \( i \) cooperates by restricting its output as long as all firms restrict their output levels in earlier periods. However, if any firm deviates even once, then firm \( i \) produces the static Cournot-Nash duopoly output level forever.

**Equilibrium in trigger strategies**

We now seek to investigate under what conditions playing trigger strategies constitutes a subgame perfect equilibrium (see Definition 2.10). It turns out that for a small discount factor, a firm may benefit by deviating from the cooperative output level, thereby collecting a temporary high profit by sacrificing the extra future profits generated by cooperation. However, for a sufficiently large discount factor we can state the following proposition:

**Proposition 6.4** If the discount factor is sufficiently large, then the outcome where both firms play their trigger strategies is a SPE. Formally, trigger strategies defined in Definition 6.4 constitute a SPE if \( \rho > 9/17 \).

**Proof.** We look at a representative period, call it period \( \tau \), and suppose that neither firm has deviated in periods \( \tau = 1, \ldots, \tau - 1 \). Then, if firm 1 deviates and plays \( q_1(\tau) = H \) (the best response to \( q_2(\tau) = L \)), Table 6.1 shows that \( \pi_1(\tau) = 9/64 > 1/8 \). However, given that firm 1 deviates, firm 2’s equilibrium strategy calls for playing \( q_2(t) = M \) for every \( t \geq \tau + 1 \). Hence the period \( \tau + 1 \) sum of discounted profits of firm \( 1 \) for all periods \( t \geq \tau + 1 \) is \( \frac{1}{1+\rho} \). Note that we used the familiar formula for calculating the present value of an infinite stream of profits given by

\[
\Pi_1 = \frac{9}{64} + \frac{\rho}{1 - \rho} \frac{1}{1}.
\]

However, if firm 1 deviates in period \( \tau \), its sum of discounted profits is

\[
\Pi_1 = \frac{9}{64} + \frac{\rho}{1 - \rho} \frac{1}{1}.
\]

(6.23)

Comparing (6.23) with (6.24) yields the conclusion that deviation is not profitable for firm 1 if \( \rho > 9/17 \).

As we noted in the proof of Proposition 2.5, to prove subgame perfection we need to show that each firm would find it profitable to respond with deviation when it realizes that deviation occurred in an earlier period, as stated in the definition of the trigger strategy described in Definition 6.4. That is, we still need to show that a firm would produce a level of \( M \) forever once either firm deviated in an earlier period. In the language of game theorists, we need to show that the trigger strategy is the best response even if the game “drifts” off the equilibrium path. However, Definition 6.4 implies that if firm \( j \) deviates, then firm \( j \) would produce \( M \) in all future periods. Then, Table 6.1 shows that firm \( i \)'s best response to firm \( j \)'s playing \( M \) is to play \( M \). Hence, the trigger strategies defined in Definition 6.4 constitute a SPE.
The purpose of section 6.5 was to demonstrate that in an infinitely repeated game, the set of oligopoly equilibria is larger than that of a one-shot game and includes cooperative outcomes in addition to the familiar noncooperative outcome. Readers who wish to learn more about cooperation in oligopolistic market structures are referred to Abreu 1986, Friedman 1971, 1977, Green and Porter 1984, Segerstrom 1988, Tirole 1988, chap. 5, and more recent books on game theory noted in the references to chapter 2.

We conclude our analysis of dynamic collusion with two remarks: (a) We have not discussed what would happen to our cooperative equilibrium when we increase the number of firms in the industry. Lamson (1984) has shown that under general demand conditions the cooperation continues to hold as long as the demand for the product increases at the same rate as the number of firms. The intuition behind this result is as follows: If the number of firms grows over time but the demand stays constant, then the future profits of each firm would drop, implying that firms would have a stronger incentive to deviate from the collusive output level. Hence, in such a case, collusion is less likely to be sustained. (b) Another natural question to be asked is how booms and recessions affect the possibility of collusion among firms. Rotemberg and Saloner (1986) analyze collusion under stochastic demand. The problem they investigate is whether collusion is more sustainable during booms (a high realization of the demand) than during recessions (a low demand realization).

### 6.6 International Trade in Homogeneous Products

In this section we analyze two issues related to international trade in homogeneous products. Subsection 6.6.1 demonstrates the possibility that countries sell homogeneous products below cost in other countries. Subsection 6.6.2 evaluates how the formation of customs unions and free trade agreements affect international trade in homogeneous products.

#### 6.6.1 Reciprocal dumping in international trade

An application of the Cournot equilibrium for international trade is given in Brander and Krugman 1983. Suppose that there are two identical trading countries indexed by \( k, k = 1, 2 \). The demand schedule in each country is given by \( p_k(Q_k) = a - bQ_k \), where \( Q \) is the sum of local production and import. In each country there is one firm producing a homogeneous product that is sold both at home and abroad. To keep this example simple, assume that production is costless, that is, \( c = 0 \).

The two countries are separated by an ocean, and therefore, shipping the good across the continents is costly. Also, assume that the transportation cost is paid by the exporting firm.

Let \( \tau \) denote the per-unit international transportation cost, and let \( q_k \) denote the production level of the firm located in country \( k, k = 1, 2 \). Since each firm sells both at home and abroad, the output of firm \( k \) is decomposed into home (local) sales (denoted by \( q^h_k \)) and foreign (export) sales (denoted by \( q^f_k \)). Therefore, the total output sold in country 1 is \( Q_1 = q^h_1 + q^f_1 \), and the total output sold in country 2 is \( Q_2 = q^h_2 + q^f_2 \).

The profit of each firm is the revenue collected in each country minus the cost of production (assumed to be zero) minus export transportation cost. Formally, the profit of the firm located in country 1 is

$$
\pi_1 = p_1(q^h_1 + q^f_1)q^h_1 + p_2(q^h_2 + q^f_2)q^f_1 - \tau q^f_1. \tag{6.25}
$$

The profit of the firm located in country 2 is

$$
\pi_2 = p_2(q^h_2 + q^f_2)q^h_2 + p_1(q^h_1 + q^f_1)q^f_2 - \tau q^f_2. \tag{6.26}
$$

The first-order conditions for (6.25) are

$$
0 = \frac{\partial \pi_1}{\partial q^h_1} = a - 2bq^f_1 - bq^h_2 \quad \text{and} \quad 0 = \frac{\partial \pi_1}{\partial q^f_1} = a - 2bq^f_1 - bq^h_2 - \tau.
$$

Notice that the two first-order conditions are independent in the sense that \( q^f_1 \) (foreign sales) does not appear in the first condition and \( q^h_1 \) (home sales) does not appear in the second. This follows from our particular use of the linear cost structure. In general, when the cost function is nonlinear, the two conditions would not be independent. The first-order conditions for (6.26)

$$
0 = \frac{\partial \pi_2}{\partial q^h_2} = a - 2bq^f_2 - bq^h_1 \quad \text{and} \quad 0 = \frac{\partial \pi_2}{\partial q^f_2} = a - 2bq^f_2 - bq^h_1 - \tau.
$$

Using this special case, we can solve for the Cournot equilibrium output levels for each country separately. In this case (6.5) implies that for firm \( k, k = 1, 2 \),

$$
q^h_k = \frac{a + \tau}{3b}, \quad q^f_k = \frac{a - 2\tau}{3b}, \quad Q_k = \frac{2a - \tau}{3b}, \quad \text{and} \quad p_k = \frac{a + \tau}{3}. \tag{6.27}
$$

Note that as transportation becomes more costly (\( \tau \) increases), the share of domestic sales increases in each country, whereas the level of export declines. Also, as \( \tau \) increases, \( p_k \) increases.
Markets for Homogeneous Products

6.6 International Trade in Homogeneous Products

given in Corden 1984 and Vousden 1990, or in almost any elementary book on international trade.

Consider the following world. There are three countries: the European Community (EC), the Far East (FE) and Israel (IL). Assume that IL is a small country, thus it cannot affect the world prices. Only FE and EC produce carpets that are imported by IL. Assume that carpets cannot be produced in IL. We further assume that IL’s demand for imported carpets is given by \( p^{IL} = a - Q \), where \( Q \) denotes the quantity demanded and \( p^{IL} \) is the domestic tariff-inclusive price.

Assume that initially (period 0), IL sets a uniform tariff of \( $t \) per carpet irrespective of where the carpets are imported from. Then, in period 1 assume that IL signs a free-trade agreement (FTA) with EC.

**Period 0:** IL levies a uniform tariff on carpets

We denote by \( p_{EC} \) the price of a carpet charged by EC’s producers, and by \( p_{FE} \) the price charged by FE’s producers. Hence, with a uniform tariff of \( t \), the price paid by IL’s consumers for carpets imported from EC is \( p^{IL}_{EC} = p_{EC} + t \), and the price paid for carpets imported from FE is \( p^{IL}_{FE} = p_{FE} + t \). We make the following assumption:

**ASSUMPTION 6.1** The export price of carpets in EC exceeds the export price in FE. Formally, \( p_{EC} > p_{FE} \).

Figure 6.4 illustrates IL’s demand for imported carpets and the prices (with and without the tariff) on carpets imported from EC and FE. Figure 6.4 shows that IL will import from the cheapest supplier, which is

**6.6.2 Homogeneous products and preferential trade agreements among countries**

There are three general types of trade agreements among countries: (1) the free-trade agreement (FTA), which is an agreement among countries to eliminate trade barriers among the member countries, but under which each country is free to set its own trade restrictions against trade with nonmember countries; (2) the customs union (CU), which is an agreement among countries to eliminate tariffs on goods imported from other member countries of the union and to set a uniform trade policy regarding nonmember countries; and (3) the common market (CM), where, in addition to the elimination of tariffs among member countries and in addition to the common tariff policy toward nonmembers, there is a free movement of factors of production among member countries.

Formal analyses of these agreements were first given by Viner, Meade and Vanek, and the interested reader is referred to surveys of literature
consumer surplus (see subsection 3.2.3 for a definition) is given by \( CS^0 = (a - p_{FE}^L)Q^0/2 \). Also, note that \( Q^0 = a - p_{FE}^L = a - p_{FE} - t \).

We define IL’s social welfare as the sum of consumer surplus plus IL’s government revenue from tariff collection. Note that in modeling international trade it is very important not to forget the existence of government’s revenue and to assume that the government returns the tariff revenue to consumers in a lump-sum fashion or by other services. Hence,

\[
W_{IL}^0 = CS^0 + G^0 = (a - p_{FE}^L + 2t)Q^0/2 = (a - p_{FE} + t)Q^0/2
\]

implying that

\[
W_{IL}^0 = \frac{(a - p_{FE} + t)(a - p_{FE} - t)}{2} = \frac{(a - p_{FE})^2 - t^2}{2}. \tag{6.29}
\]

Note that the last step in (6.29) uses the mathematical identity that \((\alpha + \beta)(\alpha - \beta) = \alpha^2 - \beta^2\). Equation (6.29) shows that the welfare of country IL decreases with the tariff rate \(t\) and with FE’s price of carpets.

**Period 1: IL signs a free-trade agreement with the EC**

Now suppose that IL signs a FTA with EC, so that the tariff on carpets imported from EC is now set to zero, whereas the tariff on imports from FE remains the same at the level of \(t\) per unit. Figure 6.5 illustrates that IL switches from importing from FE to importing from only EC for a price of \(p_{EC}^L = p_{EC}\). Given that the price of carpets drops in IL, the quantity of imported carpets increases to \(Q^1 = a - p_{EC} > Q^0\). Notice that although IL’s consumer price of carpets has decreased, IL now buys carpets from the more expensive source.

6.6 International Trade in Homogeneous Products

Under the FTA, since all the imports are from EC, the government collects zero revenue, that is \(G^1 = 0\). Hence, IL’s social welfare equals IL’s consumer surplus. That is, \(W^1 = CS^1\). The consumers’ surplus is illustrated in Figure 6.5 and is calculated to be

\[
W_{IL}^1 = CS^1 = (a - p_{EC})Q^1/2 = (a - p_{EC})^2/2. \tag{6.30}
\]

**Welfare analysis of the free-trade agreement**

We now analyze whether IL gains from the FTA with EC. Comparing (6.29) and (6.30), we see that the FTA improves IL’s welfare if \(W^1 > W^0\). That is,

\[
(a - p_{EC})^2 > (a - p_{FE})^2 - t^2
\]

or,

\[
t > \sqrt{(a - p_{FE})^2 - (a - p_{EC})^2} \tag{6.31}
\]

Therefore,

**Proposition 6.5** A free-trade agreement between IL and EC is more likely to be welfare improving for IL when (a) the initial uniform tariff is high, and (b) when the difference in prices between the two foreign exporters is small; that is, when \(p_{EC}\) is close to \(p_{FE}\).

We conclude this analysis with a graphic illustration of the gains and loss from the FTA. Figure 6.6 illustrates the welfare implication of IL’s signing the FTA with EC. In Figure 6.6, the area denoted by \(\phi\) measures IL’s consumer surplus prior to signing the FTA. The sum of the areas \(\beta + \delta\) measures IL’s government tariff revenue prior to signing the agreement. Hence, IL’s welfare prior to signing the agreement is \(W^0 = \phi + \beta + \delta\).
In Figure 6.6, the sum of the areas \( \phi + \beta + \gamma \) measures IL’s consumer surplus after the FTA is signed. Since there are no tariff revenues after the FTA (all carpets are imported from the EC), the welfare of IL after the FTA is \( W^1 = \phi + \beta + \gamma \).

Altogether, the welfare change resulting from signing the FTA is given by \( \Delta W = W^1 - W^0 = \gamma - \delta \).

**Definition 6.5** The change in consumer surplus due to the increase in the consumption of the imported good (area \( \gamma \) in Figure 6.6) is called the **trade-creation effect** of the FTA. The change in the importing country’s expenditure due to the switch to importing from the more expensive country (area \( \delta \) in Figure 6.6) is called the **trade-diversion effect** of the FTA.

Thus, the importing country gains from the FTA if the (positive) trade-creation effect associated with the increase in the import level dominates the (negative) trade-diversion effect associated to switching to importing from the more expensive source.

### 6.7 Appendix: Cournot Market Structure with Heterogeneous Firms

In this appendix we extend the analysis conducted in Subsection 6.1.2, and solve for the Cournot-market-structure equilibrium when there is a large number of firms with different cost functions. Following Bergstrom and Varian (1985), we introduce a method for calculating a Cournot-Nash equilibrium output level without resorting to solving \( N \) first-order conditions for the equilibrium \( N \) output levels.

In a Cournot market structure with \( N \) firms, each with a unit cost of \( c_i \geq 0 \), \( i = 1, \ldots, N \), each firm \( i \) chooses its output \( q_i \) that solves

\[
\max_{q_i} \pi_i(q_i, q_{-i}) = \left[ a - bq_i - b \left( \sum_{j \neq i} q_j \right) \right] q_i - c_i q_i
\]

yielding, assuming \( q_i^c > 0 \) for all \( i \), a first-order condition

\[
a - 2bq_i^c - b \left( \sum_{j \neq i} q_j^c \right) = c_i, \quad i = 1, \ldots, N.
\]

Now, instead of solving \( N \) equations (\( N \) first-order conditions) for \( N \) output levels, we solve for the aggregate production level by rewriting the first-order conditions in the form of:

\[
a - bq_i^c - bQ^c = c_i, \quad i = 1, \ldots, N.
\]

#### 6.7 Appendix: Cournot with Heterogeneous Firms

Summing over all \( q_i, i = 1, \ldots, N \) yields

\[
Na - bQ^c - bNQ^c = \sum_{i=1}^{N} c_i.
\]

Hence, the Cournot equilibrium aggregate industry output and market price are given by

\[
Q^c = \frac{Na}{(N+1)b} - \frac{\sum_{i=1}^{N} c_i}{(N+1)b} \quad \text{and} \quad p^c = \frac{a}{N+1} + \frac{\sum_{i=1}^{N} c_i}{N+1}.
\]

Hence,

**Proposition 6.6** In an industry where firms have constant unit costs, if in a Cournot equilibrium all firms produce strictly positive output levels, then the Cournot aggregate industry equilibrium output and price levels depend only on the sum of the firms’ unit costs and not on the distribution of unit costs among the firms.

The result stated in Proposition 6.6 is important, since it implies that under constant unit costs, industry output, price, and hence, total welfare can be calculated by using the sum of firms’ unit costs, without investigating the precise cost distribution among firms. Moreover, the proof of Proposition 6.6 does not rely on linear demand and therefore also applies to nonlinear demand functions.

We conclude this appendix by illustrating a simple application of Proposition 6.6. Consider an industry consisting of two type of firms: high-cost and low-cost firms. Suppose that there are \( H \geq 1 \) high-cost firms with a unit production cost given by \( c_H \), and \( L \geq 1 \) low-cost firms with a unit production cost given by \( c_L \), where \( c_H \geq c_L \geq 0 \). Substituting into (6.32) yields

\[
Q^c = \frac{(H + L)a}{(H + L + 1)b} - \frac{Hc_H + Lc_L}{(H + L + 1)b} \quad \text{and} \quad p^c = \frac{a}{H + L + 1} + \frac{Hc_H + Lc_L}{H + L + 1}.
\]

Hence, the Cournot output and price equilibrium levels depend only on \( Hc_H + Lc_L \). The advantage of learning this method for calculating Cournot equilibrium outcomes becomes clear in the case where there is an entry (or exit) of some firms. For example, suppose we observe that three additional low-cost firms have joined the industry. Then, the new Cournot equilibrium industry output and price can be immediately calculated by replacing \( Hc_H + Lc_L \) with \( Hc_H + (L + 3)c_L \) in (6.33).
6.8 Exercises

1. Two firms produce a homogeneous product. Let $p$ denote the product's price. The output level of firm 1 is denoted by $q_1$, and the output level of firm 2 by $q_2$. The aggregate industry output is denoted by $Q$, $Q \equiv q_1 + q_2$. The aggregate industry demand curve for this product is given by $p = \alpha - Q$.

Assume that the unit cost of firm 1 is $c_1$ and the unit cost of firm 2 is $c_2$, where $\alpha > c_2 > c_1 > 0$. Perform the following:

(a) Solve for a competitive equilibrium (see Definition 4.2 on page 65). Make sure that you solve for the output level of each firm and the market price.

(b) Solve for a Cournot equilibrium (see Definition 6.1 on page 99). Make sure that you solve for the output level of each firm and the market price.

(c) Solve for a sequential-moves equilibrium (see Section 6.2 on page 104) assuming that firm 1 sets its output level before firm 2 does.

(d) Solve for a sequential-moves equilibrium, assuming that firm 2 sets its output level before firm 1 does. Is there any difference in output distribution and the price level between the present case and the case where firm 1 moves first? Explain!

(e) Solve for a Bertrand equilibrium (see Definition 6.2 on page 108). Make sure that you solve for the output level of each firm and the market price.

2. In an industry there are $N$ firms producing a homogeneous product. Let $q_i$ denote the output level of firm $i$, $i = 1, 2, \ldots, N$, and let $Q$ denote the aggregate industry production level. That is, $Q \equiv \sum_{i=1}^{N} q_i$. Assume that the demand curve facing the industry is $p = 100 - Q$. Suppose that the cost function of each firm $i$ is given by

$$TC_i(q_i) = \begin{cases} F + (q_i)^2 & \text{if } q_i > 0 \\ 0 & \text{if } q_i = 0. \end{cases}$$

Solve the following problems:

(a) Suppose that the number of firms in the industry $N$ is sufficiently small so that all the $N$ firms make above-normal profits. Calculate the output and profit levels of each firm in a Cournot equilibrium.

(b) Now, assume that firms are allowed to enter or the exit from the industry. Find the equilibrium number of firms in the industry as a function of $F$. Hint: Equate a firm's profit level that you found earlier to zero and solve for $N$.

3. Consider a three-period version of the sequential-moves equilibrium analyzed in section 6.2. Assume that the market inverse demand curve is given by $p = 120 - Q$, and suppose that there are three firms that set their output levels sequentially: firm 1 sets $q_1$ in period 1, firm 2 sets $q_2$ in period 2, and firm 3 sets $q_3$ in period 3. Then, firms sell their output and collect their profits. Solve for the sequential-moves equilibrium. Make sure that you solve for the output level of each firm, and the market price.

4. Two firms compete in prices in a market for a homogeneous product. In this market there are $N > 0$ consumers; each buys one unit if the price of the product does not exceed $10$, and nothing otherwise. Consumers buy from the firm selling at a lower price. In case both firms charge the same price, assume that $N/2$ consumers buy from each firm. Assume zero production cost for both firms.

(a) Find the Bertrand equilibrium prices for a single-shot game, assuming that the firms choose their prices simultaneously.

(b) Now suppose that the game is repeated infinitely. Let $\rho$ denote the time-discount parameter. Propose trigger price strategies for both firms yielding the collusive prices of (10, 10) each period. Calculate the minimal value of $\rho$ that would enforce the trigger price strategies you proposed.

(c) Now suppose that the unit production cost of firm 2 is $4$, but the unit cost of firm 1 remained zero. Find the Bertrand equilibrium prices for the single-shot game.

(d) Assuming the new cost structure, propose trigger price strategies for both firms yielding the collusive prices of (10, 10) each period, and calculate the minimal value of $\rho$ that would enforce the trigger price strategies you propose.

(e) Conclude whether it is easier for firms to enforce the collusive prices when there is symmetric industry cost structure, or when the firms have different cost structures. Explain!

5. Consider the free-trade agreement model analyzed in subsection 6.6.2. Suppose that the world consists of three countries denoted by $A$, $B$, and $C$. Country $A$ imports shoes from countries $B$ and $C$ and does not have local production of shoes. Let the export shoe prices of countries $B$ and $C$ be given by $p_B = 60$ and $p_C = 40$. Also, suppose that initially, country $A$ levies a uniform import tariff of $t = 10$ per each pair of imported shoes. Answer the following questions:

(a) Suppose that country $A$ signs a FTA with country $B$. Does country $A$ gain or lose from this agreement? Explain!

(b) Suppose now that initially, the export price of shoes in country $C$ is $p_C = 50.01$. Under this condition, will country $A$ gain or lose from the FTA? Explain!

6. In a market for luxury cars there are two firms competing in prices. Each firm can choose to set a high price given by $p_H$, or a low price
6.9 References


Chapter 7

Markets for Differentiated Products

You can have it any color you want as long as it’s black.
—Attributed to Henry Ford

In this chapter we analyze oligopolies producing differentiated products. Where in chapter 6 consumers could not recognize or did not bother to learn the producers’ names or logos of homogeneous products, here, consumers are able to distinguish among the different producers and to treat the products (brands) as close but imperfect substitutes.

Several important observations make the analysis of differentiated products highly important.

1. Most industries produce a large number of similar but not identical products.

2. Only a small subset of all possible varieties of differentiated products are actually produced. For example, most products are not available in all colors.

3. Most industries producing differentiated products are concentrated, in the sense that it is typical to have two to five firms in an industry.

4. Consumers purchase a small subset of the available product varieties.

This chapter introduces the reader to several approaches to modeling industries producing differentiated products to explain one or more of these observations.