Electoral Competition and the Unfunding of Public Pension Programs *

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Abstract

Most public pension systems failed to build pension funds, even when it was clear that the benefits the systems were paying could not be sustained in the long run. I argue in this paper that politicians ruling public pension programs have strong incentives to exhaust the pension funds, offering generous pensions to old voters to raise the probability of winning the elections. Young voters do not support those electoral proposals to spend the pension fund, since a reduction of the fund will pull pensions down when they retire. The pension fund does not survive if old voters prevail, something that is likely to happen in the model in this paper despite of old voters being less than young voters. Electoral competition favors the elderly because they tend to be more willing to change their vote for a good pension than are young voters to change their vote for a larger pension fund.

Keywords: Electoral Competition, Pensions, Probabilistic Voting.

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1 Introduction

Most public pension systems failed to build pension funds, even when it was clear that they were becoming unsustainable in the long run. As a general rule, pension systems covered small segments of the population at the inception. The number of retirees tended to be small relative to the number of contributors during the initial years. The gradual expansion of social security coverage, with the inclusion of new contingents of contributors, helped to keep the retirees-contributors ratios low for a while. Eventually, as the systems matured, the ratios rose. In recent decades, the aging of the population has contributed to further increase these ratios. Nevertheless, neither the maturing of the pension systems nor the aging of the population necessarily imply that the ratio of benefits to contributions must deteriorate. Social Security systems can build pension funds in periods in which the pension bill is known to be temporary low in order to finance pensions when the retirees-contributors ratio becomes less favorable. However, most systems have failed to build these funds in practice, and have faced financial difficulties as a result.

The model in this paper provides an explanation for the failure of public pension systems to accumulate funds. The main hypothesis is that politicians have exhausted the pension funds, giving generous pensions to raise their probability of winning elections. This is a model of a representative democracy in which citizens must choose among two competing office-seeking candidates. In order to win the elections, the candidates make (binding) promises in several fronts, including pensions for the currently old citizens. Candidates cannot credibly offer good pensions to the currently young voters, because those pensions will be decided in the future by other politicians, after new elections. What current candidates can offer to woo young voters is not to spend the pension fund, leaving the pension system administration in a better position to grant good pensions in the future. Therefore, while old voters push politicians towards more spending, young voters do the opposite. The pension fund does not survive if old voters prevail, something that is likely to happen in this model despite of old voters being less than young voters.

The advantageous position of the elderly in this paper comes from the asymmetric ability of politicians to channel transfers to young and old citizens through the pension system. Politicians can more effectively gain votes from old citizens granting better current pensions than from young citizens preserving the pension fund. Young voters could be interested in the pension fund if they thought that a large fund warrants a good pension for them. But if young voters saw that the current administration preserves the fund paying modest pensions, they should think that the following administration could do the same, in which case the fund would not benefit them. Therefore, young voters may not be especially interested in the pension fund, failing to "discipline" politicians to preserve the fund.

The idea that old voters are more responsive than young voters to pension issues in terms of votes looks consistent with the facts. Mulligan and Sala-i-Martin (1999) put it in this way: "The most important concern among elderly voters are government old age subsidies and is believed by many politicians that the votes of the elderly are much more elastic to a candidate's stance on old age subsidies than are the votes of any other group to any other issue". The model in this paper provides an explanation of why this could be so, and explores its consequences for the working of the pension system.

The politico-economic equilibria described in this paper show that a pension system administration driven by electoral competition is likely to exhaust the fund during its first years and to pay decreasing pensions to successive generations. This theory is consistent with the evolution of actual institutions in many countries. Most public pension systems are currently fully unfunded, but not all of them were initially so. Nevertheless, one way or another, accumulated reserves were progressively eroded (Disney,1996, p 59; World Bank, 1994; Mulligan, 2000). German pension funds lost their assets during the world wars. Greece, Italy, Portugal, Spain, and Turkey granted generous pensions that were unsustainable in the long run (Disney 1996, p 85). In the US, Nixon rose pensions by twenty per cent just ahead of the 1972 elections, sending a letter to retirees to let them be aware of his generosity (Tufte, 1978; Woolley 1986, ver también Drazen 2000, pp 231-2). Some Latin American countries extended benefits to new segments of the population without previously requiring contributions from the new beneficiaries.[1]

Fears of political misuse of the trust fund are currently informing the political debate on Social Security reform in the United States. Critics of the "advance funding" proposal in anticipation of future solvency problems

[1] In terms of Mesa-Lago and Bertranou (1998, p 27): "Political parties have also created new programs or liberalized the existing ones to the covered population. These concessions have usually preceded national elections: the party in power has passed legislation to get popular support and the parties in the opposition have promised those changes if they are elected." (Translated from the original Spanish version).

argue that the Congress will likely use the fund to increase benefits or reduce taxes or spend them for other purposes (Diamond, 1999, p 99; Mulligan, 2000). Munnell (1998) has proposed a complete separation of the Social Security budget from the rest of the budget. However, as Alesina (2000) points out "this step might avoid using the Social Security surplus for discretionary spending, but it would not avoid increasing Social Security benefits for current generations of voters at the expense of future generations". This is precisely what the politicians ruling Social Security do in the model presented in this paper.

Existing theories of electoral competition in social security have adopted a majority voting framework (Browning 1975, Hu 1982, Boadway and Wildasin 1989, Tabellini 1991, Casamatta, Cremer, and Pestieau 2000). However, in modern representative democracies policies are usually not decided by direct majority voting by the citizenship, as the majority voting model assumes (Tullock 1998). Citizens vote for political parties that represent them in many different dimensions, some of which become apparent only after elections. The promises candidates make on social security, even if binding, refer to just one of the many issues involved in an election. Probabilistic voting represents this decision process more accurately than majority voting. More importantly, the choice of the model matters because the outcome of these models is generally different. While in majority voting politicians please the median voter, in a probabilistic voting environment political parties must please the mobile voters, those that are more willing to exchange votes for economic benefits (Persson and Tabellini 2000).

The assumption of a direct vote for the pay-as-you-go pension system has faced the majority voting models of social security with the challenge of explaining why the median voter, who is typically an active worker, would vote for a program that favors the retirees. According to Mulligan and Sala-i-Martin (1999), these theories include one of the following additional hypotheses to deal with this difficulty: a) the elderly ally themselves with some poor young voters (Tabellini 1991), b) there is only one election in which the vote is for a stationary policy (Browning 1975). However, these additional hypotheses confront problems. Mulligan and Sala-i-Martin argue that the idea of a winning coalition of the elderly and the poor is to a large extent imposed: other coalitions could be formed with equal chances of winning. In turn, Browning's model is not robust to "temporary suspension": young and middle-aged voters would conform a majority voting for a suspension of the transfers to the old for one period. The same logic would drive to a suspension in the following periods, however, and the system would never get political support.

The probabilistic voting model does not face the problem considered in the previous paragraph. While the decisive voter in majority voting is an active worker that may not be particularly interested in favoring the elderly, old voters may become decisive in probabilistic voting. Old voters tend to be more responsive to offers related to pensions than young voters because politicians are more able to commit pensions in the near than in the far future. The most a candidate can offer to please young voters is to observe fiscal discipline, abstaining from giving too generous pensions to the currently old citizens. Therefore, while old citizens must be very sensitive to current pension issues, young voters only care about them when the situation becomes critical and major reforms are being analyzed. In normal times, young citizens base their votes on other issues.

2 The argument

The main hypothesis of the paper is that politicians exhaust the pension funds giving generous pensions to raise their probability of winning the elections. This is a redistributive decision that favors the relatively few old voters at the expense of the many young voters. This behavior of politicians is not consistent with Downsian electoral competition, but it is consistent with probabilistic voting. Hence, using a sketchy model of probabilistic voting and drawing on previous work by Dixit and Londregan (1996), I explore in this section several reasons why office-seeking politicians might redistribute in favor of some particular groups of voters, and show that the number of members of the group is not one of these reasons. I argue then that old voters have some of the characteristics that make a group of citizens successful in the redistributive process. In particular, I argue that politicians are more efficient in channeling resources to old than to young citizens through the pension system, and that this makes them a good target for electoral-motivated redistribution.

Consider a society composed of two groups, the old and the young citizens, identified with subindex $i = \{o, y\}$, respectively. Each member of group i has an income w_i, and receives public transfers $t_i \geq -w_i$. Their income preferences can be represented by continuous, increasing and concave utility functions $u_i(w_i + t_i)$. The number of old and young citizens is N_y and N_o,respectively. The government faces the following budget constraint: $N_o t_o (1 + \alpha_o) + N_y t_y (1 + \alpha_y) = 0$, where the parameters $\alpha_i \geq 0$ capture the cost of the redistributive process (the government spends $(1 + \alpha_i)$ to give citizen i a transfer of one unit). There are two political parties, A and B, whose only goal is to maximize their vote share in the elections.[2] Political candidates can commit electoral platforms t_o^A and t_o^B (notice that the policy space is unidimensional in this example with only two groups: the transfers to the old determine the transfers to the young, because of the government budget constraint). I will not endogenize the decision to participate in the elections, but I will allow for different turnout rates among the old and the young citizens ($0 \leq e_i \leq 1$), with the only constraint that the number of old voters is smaller than the number of young voters: $e_o N_o < e_y N_y$.

Suppose first, in the spirit of Downs (1957), that citizens decide their vote based solely on the candidates' proposed policies. Citizen i votes for candidate A if $u_i\left(w_i + t_i^A\right) > u_i\left(w_i + t_i^B\right)$, for B if $u_i\left(w_i + t_i^A\right) < u_i\left(w_i + t_i^B\right)$, and flips a coin if $u_i\left(w_i + t_i^A\right) = u_i\left(w_i + t_i^B\right)$. Given that young voters outnumber old voters, office-seeking candidates will try to please young voters. The candidate that offers them a more appealing electoral platform get the majority of votes.[3] Hence candidates will propose $t_o^A = t_o^B = -w_o$ and $t_y^A = t_y^B = N_o w_o (1 + \alpha_o) / [N_y (1 + \alpha_y)]$. The old voters are the losers in this case.

Suppose now, following Lindbeck and Weibull (1988), that citizens care about candidates' policies, but also have some partisan preferences that do not depend on electoral platforms. More specifically, assume that citizen i's preference for B relative to A can be represented by the utility index σ_i that can be added to $u_i(.)$ to get total utility. Citizen i votes for A if $u_i\left(w_i + t_i^A\right) > u_i\left(w_i + t_i^B\right) + \sigma_i$. Both groups have voters with different partisan preferences. For simplicity, assume that the distribution of σ_i in each group is uniform with density h_i and zero mean. Politicians do not know each voter's partisan preference, but they know whether an individual voter is young or old and the distribution of the partisan preferences in both

[2] The same results hold if politicians are assumed to maximize the probability of winning the elections, but the model becomes slightly more complex. For the sake of simplicity, I assume in this section that policitians maximize their vote share rather than the probability of winning.

[3] The median voter result holds in this example, for preferences are single peaked. Also, the median voter is young if young are more than old voters.

groups.

When the election comes, citizens with partisan preference $\sigma_i < u_i\left(w_i + t_i^A\right) - u_i\left(w_i + t_i^B\right)$ vote for A. Hence, the number of votes for A is:

$$V^A = \sum_{i \in \{o,y\}} N_i e_i \left\{ \frac{1}{2} + h_i \left[u_i\left(w_i + t_i^A\right) - u_i\left(w_i + t_i^B\right) \right] \right\}$$

and the number of votes for B is: $V^B = e_o N_o + e_y N_y - V^A$.

During the electoral campaign, candidates simultaneously choose their electoral platforms to maximize their vote share. Candidate j's best response to the other candidate platform is the solution of the following program:

$$\begin{aligned} &\underset{t_o^j, t_y^j}{Maximize} \quad V^j \\ &st: \quad N_o t_o^j \left(1 + \alpha_o\right) + N_y t_y^j \left(1 + \alpha_y\right) = 0 \end{aligned}$$

The first order conditions of this program imply that:[4]

$$\frac{u_y'\left(w_y + t_y^j\right)}{u_o'\left(w_o + t_o^j\right)} = \left(\frac{1 + \alpha_y}{1 + \alpha_o}\right) \frac{h_o}{h_y} \frac{e_o}{e_y} \tag{1}$$

$$N_o t_o^j \left(1 + \alpha_o\right) + N_y t_y^j \left(1 + \alpha_y\right) = 0 \tag{2}$$

In a Nash equilibrium, both candidates choose the transfers that solve 1-2.

I will consider several combinations of the parameters of this society to see which are the driving forces behind the redistributive process in this environment.

Case 1: The benchmark case.

Assumptions:

(A1) The government is equally efficient in redistributing to both groups of citizens: $\alpha_o = \alpha_y$.

(A2) Equal turnout rates: $e_o = e_y$.

(A3) Old and young citizens have the same partisan preferences: $h_o = h_y$.

(A4) Equal income before transfers: $w_o = w_y = w$.

(A5) Equal utility function: $u_o(x) = u_y(x) = u(x)$.

[4] It is straightforward to check that the second order conditions hold.

Assumptions (A1)-(A5) and equations 1-2 imply that $t_o = t_y = 0$. [5] Unlike in the Downsian model of redistribution, young voters have no advantage over old voters in this model, despite of being more and having the median voter among them.

Case 2: Old citizens are more sensitive to transfers than young citizens.

Assumptions: (A1)-(A4), and:

(A5') $u_o(x) = \gamma u_y(x)\,, \ \ \gamma > 1.$

These assumptions imply that:

$$\begin{aligned} \frac{u_y'(w+t_y)}{u_o'(w+t_o)} &= \frac{u_y'(w+t_y)}{\gamma u_y'(w+t_o)} = 1 \\ N_o t_o + N_y t_y &= 0 \end{aligned}$$

and hence $t_y < 0 < t_o$. Therefore, old citizens benefit from electoral competition, if they are more sensitive to transfers or more "apolitical" than young citizens.

Case 3: Old citizens are poorer than young citizens.

Assumptions: (A1), (A2), (A3), (A5), and:

(A4') $w_o < w_y$. $\Rightarrow$

$$\begin{aligned} &w_y - w_o = t_o - t_y > 0 \\ &N_o t_o + N_y t_y = 0 \end{aligned}$$

and hence $t_y < 0 < t_o$. Politicians favor poorer voters because they are more responsive to transfers in terms of votes.

Case 4: Old citizens are politically more "centrists" than young citizens.

Assumptions: (A1), (A2), (A4), (A5), and:

(A3') $h_o > h_y$. $\Longrightarrow$

$$\begin{aligned} \frac{u'(w+t_y)}{u'(w+t_o)} &= \frac{h_o}{h_y} > 1 \\ N_o t_o + N_y t_y &= 0 \end{aligned}$$

and hence $t_y < 0 < t_o$. A group that has more citizens in the political center is more responsive to policy proposals, and becomes a better target for electoral motivated redistribution.

[5] Both candidates propose the same platform in equilibrium (full convergence of policies), and hence there is no need to keep the superindex.

Case 5: Old citizens have a larger turnout rate than young citizens.[6]
Assumptions: (A1), (A3), (A4), (A5), and:
(A2') $e_o > e_y$. $\Longrightarrow$

$$\begin{aligned} \frac{u'(w+t_y)}{u'(w+t_o)} &= \frac{e_o}{e_y} > 1 \\ N_o t_o + N_y t_y &= 0 \end{aligned}$$

and hence $t_y < 0 < t_o$. The resources that a politician has to spend to get an aditional vote rises with abstentionism, because transfers cannot be channeled only to those who vote. Therefore, politicians find it more costly to gain the votes of groups with more abstentionism and prefer to spend more resources on groups with larger turnout rates. Notice the difference with the number of members of the group. There are many votes involved in large groups, but the cost of getting each vote is not necessarily smaller in those groups, and hence politicians are not willing to give larger per capita transfers to members of large groups.

Case 6: The government is more efficient in giving transfers to the old than to the young citizens.

Assumptions: (A2), (A3), (A4), (A5) and:
(A1') $\alpha_o < \alpha_y$ $\Longrightarrow$

$$\begin{aligned} &\frac{u'_y(w_y+t_y)}{u'_o(w_o+t_o)} = \left(\frac{1+\alpha_y}{1+\alpha_o}\right) > 1 \\ &N_o t_o(1+\alpha_o) + N_y t_y(1+\alpha_y) = 0 \end{aligned}$$

implying that $t_y < 0 < t_o$. If the government can more efficiently channel resources to old than to young voters, politicians find it "cheaper" to gain votes from old than from young citizens.

The benchmark case does not predict any transfers, and cannot thus explain why old citizens might benefit from intergenerational redistribution through the pension system. It does show that old citizens do not lose either, even though they are a minority in the electorate. The following five cases predict that old citizens benefit at the expense of young citizens, providing potential explanations for the intergenerational redistribution this paper is aiming to explain. Case 2 shows that old voters would be the winners if they were more "apolitical" than young voters. But it is quite doubtful

[6] I am in debt with Juan Dubra for drawing my attention to this issue.

that real world senior voters are systematically less politicized than young voters. Case 3 shows that politicians would redistribute in favor of old voters if they were poorer than young voters, something that looks consistent with economic growth. Early advocators of public and universalistic social security emphasized the role of the pension system in alleviating poverty among the elderly. The analysis in this section suggests that office-seeking politicians had reasons to support these proposals. According to the analysis in case 4, politicians would benefit old voters if they were more "centrists" than young voters. I am not aware of any empirical evidence supporting this assumption. To be continued...

3 The model

3.1 Description of the society

The economy is small and open. Capital moves freely across frontiers and interest rate parity holds. Hence, the domestic interest rate (R) is equal to the international rate, which I assume constant for simplicity.

Production is carried on by a large number of competitive firms that combine labor and capital to produce output. The technology exhibits constant returns to scale.

The society is populated by citizens who live two periods. N_{t-1} old citizens and N_t young citizens live in period t. I will assume, for simplicity, that the rate of growth of the population is constant: $n = (N_t/N_{t-1}) - 1$. Individuals work during their first period of life and are retirees during the second. In period t, a young citizen earns the pre-tax wage $w \geq 0$, pays a fraction $\tau_t \in [0, 1]$ to the social security administration and expects to receive the pension $p_{t+1} \geq 0$ when he is old. Hence, a member of generation t has life-time income $\omega_t = w(1 - \tau_t)(1 + R) + p_{t+1}$. His preferences among young (c_{yt}) and old age (c_{ot+1}) consumption can be represented by a separable utility function: $u(c_{yt}) + \beta u(c_{ot+1})$, where $u(.)$ is a continuous, increasing, and concave function, and β is a positive parameter representing the subjective discount factor. Let $U(\omega_t)$ be the associated indirect utility.

In t, politicians ruling the pension system choose pensions to be paid to the currently old, taxes to be collected from young citizens and a pension fund to be left for the following period. Let a_{t+1} denote the pension fund per member of generation $t+1$ at the beginning of period $t+1$. The policy choice of the authorities in t can be summarized by the vector $T_t = (p_t, \tau_t, a_{t+1})$. A *feasible policy* is a sequence $\{T_t\}_{t\geq 0}$ such that:

$$a_t (1+n)(1+R) + (1+n) w\tau_t - p_t = a_{t+1} (1+n)^2 \ , \ t \geq 0 \tag{3}$$

and

$$a_t \geq 0 \ , \ t \geq 0 \tag{4}$$

The main results in this paper do not change qualitatively if the constraint 4 is relaxed allowing the system to build some debt, i.e. to have a_t negative, provided it has a lower bound. This constraint is more stringent than the no-Ponzi-game condition that is often imposed on governments, but it looks realistic.

The per-period budget constraints of the pension system give rise to the following intertemporal budget constraint:

$$\sum_{t=0}^{\infty} [\omega_{t-1} - w(1+R)] \left(\frac{1+n}{1+R}\right)^t = \lim_{T\to\infty} \left[w(1+n)\tau_T - a_{T+1}(1+n)^2\right] \left(\frac{1+n}{1+R}\right)^T \tag{5}$$

The fact that the tax rate is bounded above ($\tau_T \leq \tau$) and the pension fund is bounded below ($a_{T+1} \geq 0$), imply that in a dynamically efficient economy (i.e. when $R > n$), the intertemporal budget constraint 5 boils down to:

$$\sum_{t=0}^{\infty} [\omega_{t-1} - w(1+R)] \left(\frac{1+n}{1+R}\right)^t \leq 0 \ \ , \ \ if \ \ R > n \tag{6}$$

Elections take place at the beginning of every period. Two political candidates compete in each election, with no candidate participating in more than one campaign. Their goal is to maximize the probability of winning the election.[7] During the electoral campaign for period-t elections, candidate $i = A, B$ announces the policy $T_t^i = \left(p_t^i, \tau_t^i, a_{t+1}^i\right)$. Citizens consider these electoral platforms in deciding their vote, because politicians are assumed to be able to commit to the promises made in the electoral campaign.[8] Let

[7]The results do not change if political parties are assumed to maximize the number of votes rather than the probability of winning the elections (Lindbeck and Weibull, 1987; Dixit and Londregan, 1996; Persson and Tabellini, 2000, p 177).

[8]Office-motivated politicians have no incentives to do after elections anything different from what they promised before elections. The commitment assumption is more controversial when politicians are motivated by the outcome of policies.

$t = 0$ be the first period in which politicians are able to credibly propose to organize a public pension system. The system actually begins in this period if the candidate that wins period-0 elections announced non-zero contributions ($\tau_0 > 0$). Having no pension system in previous periods, the pension fund at the beginning of period zero is null ($a_0 = 0$).

Citizens well being depends on both pension policies and some permanent attributes of the elected politician. Old citizens preferences over current electoral platforms are easy to derive from primitive assumptions on consumption preferences, but things are less simple in the case of young citizens. Their indirect utility depends not only on policies chosen by politicians elected in current elections but also on policies to be chosen in the following period after another election. Rational young voters take into account the effects that current choices may have on next period policies. Nevertheless, I show in the next section that, if candidates play Markov strategies, citizens have well defined order of preferences over T_t^i that can be represented by continuous *functions of policy preferences* $U_o(T_t^i)$ and $U_y(T_t^i)$ for old and young citizens, respectively.

Citizens care about other attributes of political parties as well, so they have *partisan preferences*. There is no unanimity in this respect. Following previous literature (Persson and Tabellini 2000), I assume that partisan preferences for B relative to A have an individual-specific component ($\sigma \in R$) and a whole-population component ($\delta \in R$); the former is variable across individuals and the latter is a parameter of the population. Total utility is assumed to be the sum of the partisan preference parameters and the function of policy preferences. In t, old citizens get utility $U_o\left(T_t^B\right) + \sigma + \delta$, if B is in office, and $U_o\left(T_t^A\right)$, if A is in office. Young citizens get utility $U_y\left(T_t^B\right) + \sigma + \delta$, if B is in office, and $U_y\left(T_t^A\right)$, if A is in office.

The partisan preference parameters are not generally known with certainty. Each citizen knows his own partisan preference ($\sigma + \delta$), without being able to distinguish between σ and δ. The cumulative distribution function $F(\delta)$ represents individuals believes about δ, and the cumulative distribution function $H(\sigma)$ represents the distribution of the individual-specific parameter in the population. These distributions are time invariant.

The timing in each period is as follows: a) Political parties choose electoral platforms T_t^i. b) Elections take place. c) The winning party implements the announced policy, firms produce, and citizens choose consumption. These three steps repeat period after period from $t = 0$ onwards.

3.2 The politico-economic equilibrium

Production. After the elections, firms choose the capital-labor ratio that minimizes costs. The optimum capital-labor ratio is a function of the international interest rate. In equilibrium, wages must be equal to the marginal product of labor evaluated at the optimum capital-labor ratio. The assumption of a constant international interest rate implies that the capital-labor ratio and the wage are constant.

The functions of policy preferences with Markov policies. During period t, old citizens maximize their utility choosing as large as possible consumption: $c_{ot} = s_{t-1}(1+R) + p_t$, where young-age savings (s_{t-1}) are a given at this stage. Hence, the function of policy preferences for old citizens is:

$$U_o(T_t) = u(s_{t-1}(1+R) + p_t)$$

Young citizens choose consumption solving the following program:

$$\begin{aligned} U(\omega_t) &= \underset{Cyt,Cot+1}{Maximize}\ u(c_{yt}) + \beta u(c_{ot+1}) \\ st &: c_{yt}(1+R) + c_{ot+1} \leq \omega_t = w(1-\tau_t)(1+R) + p_{t+1} \end{aligned} \tag{7}$$

Young citizens well being depends on policies implemented in two successive periods. The indirect utility of a member of generation t is a function of his life-time income ω_t, which is in turn a function of policies implemented by the authorities of the pension system in t and $t+1$. Hence, in period t members of generation t have policy preferences that can be represented by a function $U(\omega_t(T_t, T_{t+1}))$. They have to take some decisions already in t before knowing policy T_{t+1}. Notwithstanding, citizens can take advantage of their knowledge of the structure of the game to compute next period policies.

In principle, the future government can choose any feasible T_{t+1} and have it depend on the whole history of the game up to period $t+1$. However, it looks sensible in this model to restrict the attention to Markov policies, i.e. to policy vectors that depend only on the "pay-off relevant" history. More especifically, I will assume that young citizens base their vote in t on the conjecture that next period candidates will propose pensions that depend only on the pension fund at the beginning of $t+1$ and the taxes paid in t:

$p^i_{t+1}(\tau_t, a_{t+1})$. These two variables summarize all the information about the past that has any "direct" bearing on payoffs in $t+1$ and following periods.[9] Of course, this conjecture must prove ex-post correct in equilibrium. Also citizens know that the two candidates will propose the same policy in equilibrium for office motivated politicians have no incentives to differentiate their policies, i.e. citizens rationally expect "full convergence" of the electoral platforms. Hence, citizens expect $p^A_{t+1}(\tau_t, a_{t+1}) = p^B_{t+1}(\tau_t, a_{t+1}) = p^*_{t+1}(\tau_t, a_{t+1})$. With the Markov assumption, the best responses of next period candidates to current policies can be written as $T^*_{t+1}(T_t) = \left(p^*_{t+1}(\tau_t, a_{t+1}), \tau^*_{t+1}, a^*_{t+2}\right)$, implying that members of generation t have preferences over period-t policies as follows:

$$U_y(T_t) = U\left(\omega_t\left(T_t, T^*_{t+1}(T_t)\right)\right) = U\left(w(1-\tau_t)(1+R) + p^*_{t+1}(\tau_t, a_{t+1})\right)$$

Elections Given the electoral platforms, young citizens with individual-specific parameter σ_{yt} and old citizens with individual-specific parameter σ_{ot} are indifferent between candidates A and B in period-t elections, if:

$$\sigma_{yt} = U_y\left(T^A_t\right) - U_y\left(T^B_t\right) - \delta \quad (8)$$

$$\sigma_{ot} = U_o\left(T^A_t\right) - U_o\left(T^B_t\right) - \delta \quad (9)$$

Thus, candidate A receives the vote of young citizens with partisan preferences $\sigma < \sigma_{yt}$ and old citizens with partisan preferences $\sigma < \sigma_{ot}$. The total number of votes for A in period-t elections is:

$$V^A_t\left(T^A_t, T^B_t, \delta\right) = N_{t-1}H(\sigma_{ot}) + N_t H(\sigma_{yt}) \quad (10)$$

and the number of votes por B is:

$$V^B_t\left(T^A_t, T^B_t, \delta\right) = N_{t-1} + N_t - V^A_t \quad (11)$$

[9] (τ_t, a_{t+1}) summarizes the minimal partition of the history of the game up to period $t+1$ that is sufficient in the sense that all histories that have reached this point have: a) the same feasible policies from $t+1$ onwards, and b) for all citizens, the utility functions conditional on these histories are representations of the same preferences. (See Fudenberg and Tirole, 1992, for a general presentation of the concept of payoff-relevant history).

Let $\delta\left(T_t^A, T_t^B\right)$ be the value of δ such that A gets exactly half of the votes when the electoral platforms are $\left(T_t^A, T_t^B\right)$:[10]

$$V_t^A\left(T_t^A, T_t^B, \delta\left(T_t^A, T_t^B\right)\right) = \frac{1}{2}\left(N_{t-1} + N_t\right) \tag{12}$$

Candidate A wins period-t elections if $\delta < \delta\left(T_t^A, T_t^B\right)$, and hence the probability that A wins this election is

$$G^A\left(T_t^A, T_t^B\right) = F\left(\delta\left(T_t^A, T_t^B\right)\right)$$

and the probability that B wins this elections is: $G^B\left(T_t^A, T_t^B\right) = 1 - G^A\left(T_t^A, T_t^B\right)$.

Electoral platforms Before elections, candidates choose their platforms to maximize the probability of winning the elections:

$$\underset{T_t^i}{Maximize}\ G^i\left(T_t^i, T_t^j\right) \quad , \ i,j = A,B \tag{13}$$

The solution of this optimization program is the best-response correspondence of candidate i to candidate j platform: $T_t^i\left(T_t^j\right)$. In a politico-economic equilibrium, the electoral platform of candidate i must be a fixed point of the mapping: $T_t^i\left(T_t^j\left(T_t^{i*}\right)\right) = T_t^{i*}$.

Definition 1 *a politico-economic equilibrium is a sequence of feasible Markov policies* $\left\{\left(T_0^{A*}(0), T_0^{B*}(0)\right), ..., \left(T_t^{A*}\left(T_{t-1}^*\right), T_t^{B*}\left(T_{t-1}^*\right)\right), ...\right\}$*, feasible consumption vectors* $\left\{c_{o0}^*, \left(c_{y0}^*, c_{o1}^*\right), ..., \left(c_{yt}^*, c_{ot+1}^*\right), ...\right\}$*, and citizens votes such that: a)* $G^i\left(T_t^{i*}\left(T_{t-1}^*\right), T_t^{j*}\left(T_{t-1}^*\right)\right) \geq G^i\left(T_t^i\left(T_{t-1}^*\right), T_t^{j*}\left(T_{t-1}^*\right)\right)$, $i,j = A,B$, $t \geq 0$, *where* $T_t^i\left(T_{t-1}^*\right)$ *are arbitrary feasible Markov policies; b)* $u\left(c_{o0}^*\right) \geq u\left(c_{o0}\right)$ *and* $u\left(c_{yt}^*\right) + \beta u\left(c_{ot+1}^*\right) \geq u\left(c_{yt}\right) + \beta u\left(c_{ot+1}\right), t \geq 0,$ *where* c_{o0}, c_{yt}, *and* c_{ot+1} *are arbitrary feasible consumption values, and c) in period-t elections, members of generation t vote for* $A(B)$ *if*

$$U_y\left(T_t^{A*}\right) > (<)\, U_y\left(T_t^{B*}\right) + \sigma + \delta,$$

and flip a coin otherwise; and members of generation $t-1$ *vote for* $A(B)$ *if*

$$U_o\left(T_t^{A*}\right) > (<)\, U_o\left(T_t^{B*}\right) + \sigma + \delta,$$

and flip a coin otherwise.

[10] Equation 12 defines a continuum of δ if $\partial V_t^A / \partial \delta = 0$ when $V_t^A(.) = \left(N_{t-1} + N_t\right)/2$. In this case, define $\delta\left(T_t^A, T_t^B\right)$ as the minimum δ for which 12 holds true.

The existence of the equilibrium is not generally warranted with the assumptions I have made so far. The main difficulty stems from the fact that the maping $G^i\left(T_t^i, T_t^j\right)$ may not be sufficiently "well behaved". Quasi-concavity of $G^i\left(T_t^i, T_t^j\right)$ in T_t^i would be sufficient to have best-response correspondences such that Kakutani's fixed point theorem would apply, but this condition is not warranted without further restricting the individuals primitive utility functions and the distribution functions of the partisan preference parameters. Rather than addressing the existence problem, I will first assume that an equilibrium with continuous and differentiable policy maps exists and analyze its properties. Then I will consider a parametric example in which the equilibrium does exist.

4 Characteristics of equilibria

The equilibria are symmetric in this model, in the sense that both parties propose the same policies in equilibrium. This is not a particular feature of the present model, but a common characteristic of models of electoral competition in which politicians do not have policy preferences (Lindbeck and Weibull 1987, Persson and Tabellini 2000, Roemer 2001, among others). Hence, I will not prove this full-convergence result here, and I proceed by noting that $T_t^{A*} = T_t^{B*}$ and equations 8 and 9 imply that young and old voters that are indifferent between A and B in equilibrium must have the same partisan preferences:

$$\sigma_{yt}^* = \sigma_{ot}^* = -\delta \tag{14}$$

I argued in section 2 that the ability of policitians to channel resources to different groups is a key determinant of their performance in the redistributive process. In redistribution taking place through the pension system, politicians can directly give old voters pensions, while only indirectly can they favor young voters leaving a positive pension fund. Hence, politicians ability to benefit young voters crucially depends on the sensitivity of next period pensions to the size of the fund that the current administration leaves. The following proposition shows that the larger is the the sensitivity of next period pensions to the pension fund, the better young voters are and the worse old voters are in an equilibrium with positive pension fund. This proposition also establishes that the pension fund does not survive if agents think that next period pensions are not an increasing function of the size of the pension fund at the beginning of next period.

Proposition 1 *The equilibrium path with positive pensions verifies:*

$$\frac{u'(c_{ot})}{\beta u'(c_{ot+1})}\begin{cases} = \frac{1}{1+n}\partial p_{t+1}/\partial a_{t+1} & , \; if \quad a_{t+1}>0 \\ \geq \frac{1}{1+n}\partial p_{t+1}/\partial a_{t+1} & , \; if \quad a_{t+1}=0 \end{cases} \tag{15}$$

Proof. In order to characterize the equilibrium policies, I write candidates optimization program 13 more explicitly. Candidate A solves the following program: [11]

$$\begin{aligned} & \underset{p_t^A,\tau_t^A,a_{t+1}^A}{Maximize} \quad F\left(\delta\left(p_t^A,\tau_t^A,a_{t+1}^A,T_t^B\right)\right) \\ & st: \\ & a_t(1+n)(1+R)+(1+n)w\tau_t^A-p_t^A \geq a_{t+1}^A(1+n)^2 \\ & \tau_t^A \leq \tau \;\; , \quad 0<\tau\leq 1 \\ & p_t^A \geq 0,\; \tau_t^A \geq 0,\; a_{t+1}^A \geq 0 \end{aligned}$$

The Lagrangean is:

$$\begin{aligned} & L^A = F\left(\delta\left(p_t^A,\tau_t^A,a_{t+1}^A,T_t^B\right)\right) \\ & +\lambda_1^A\left[a_t(1+n)(1+R)+(1+n)w\tau_t^A-p_t^A-a_{t+1}^A(1+n)^2\right] \\ & +\lambda_2^A\left(\tau-\tau_t^A\right) \end{aligned}$$

and the Kuhn-Tucker conditions are:

$$\frac{\partial L^A}{\partial T_t^A}\left(T_t^{A*},\lambda_1^{A*},\lambda_2^{A*}\right) \leq 0 \tag{16}$$

$$\frac{\partial L^A}{\partial \lambda_1^A}\left(T_t^{A*},\lambda_1^{A*},\lambda_2^{A*}\right) \geq 0 \tag{17}$$

$$\frac{\partial L^A}{\partial \lambda_2^A}\left(T_t^{A*},\lambda_1^{A*},\lambda_2^{A*}\right) \geq 0 \tag{18}$$

$$T_t^{A*}\frac{\partial L^A}{\partial T_t^A}\left(T_t^{A*},\lambda_1^{A*},\lambda_2^{A*}\right) = 0 \tag{19}$$

$$\lambda_1^A\frac{\partial L^A}{\partial \lambda_1^A}\left(T_t^{A*},\lambda_1^{A*},\lambda_2^{A*}\right) = \lambda_2^A\frac{\partial L^A}{\partial \lambda_2^A}\left(T_t^{A*},\lambda_1^{A*},\lambda_2^{A*}\right)=0 \tag{20}$$

$$T_t^{A*} \geq 0, \quad \lambda_1^{A*}\geq 0, \quad \lambda_2^{A*}\geq 0 \tag{21}$$

[11] Candidate B solves a symmetric problem.

where $\partial L^A/\partial T_t^A$ is the gradient column vector 3x1:

$$\partial L^A/\partial T_t^A = \begin{pmatrix} \partial L^A/\partial p_t^A \\ \partial L^A/\partial \tau_t^A \\ \partial L^A/\partial a_{t+1}^A \end{pmatrix} = F'(.)\,\partial\delta/\partial T_t^A + \begin{pmatrix} -\lambda_1^{A*} \\ \lambda_1^{A*}(1+n) - \lambda_2^{A*} \\ -\lambda_1^{A*}(1+n)^2 \end{pmatrix}$$

which can be written more explicitly as:

$$\frac{\partial L^A}{\partial p_t^A}\left(T_t^{A*}, \lambda_1^{A*}, \lambda_2^{A*}\right) = F'(.)\frac{\partial\delta}{\partial p_t^A}(.) - \lambda_1^{A*} \quad (22)$$

$$\frac{\partial L^A}{\partial \tau_t^A}\left(T_t^{A*}, \lambda_1^{A*}, \lambda_2^{A*}\right) = F'(.)\frac{\partial\delta}{\partial \tau_t^A}(.) + \lambda_1^{A*}w(1+n) - \lambda_2^{A*} \quad (23)$$

$$\frac{\partial L^A}{\partial a_{t+1}^A}\left(T_t^{A*}, \lambda_1^{A*}, \lambda_2^{A*}\right) = F'(.)\frac{\partial\delta}{\partial a_{t+1}^A}(.) - \lambda_1^{A*}(1+n)^2 \quad (24)$$

The function $\delta\left(p_t^A, \tau_t^A, a_{t+1}^A, T_t^B\right)$ is implicitly defined by equations 8, 9, 10 and 12. Therefore:

$$\frac{\partial\delta}{\partial T_t^A}(.) = \frac{N_{t-1}H'(\sigma_{ot})\frac{\partial U_o}{\partial T_t^A}\left(T_t^A\right) + N_t H'(\sigma_{yt})\frac{dU_y}{dT_t^A}\left(T_t^A\right)}{\left[N_{t-1}H'(\sigma_{ot}) + N_t H'(\sigma_{yt})\right]}$$

and:

$$\frac{\partial\delta}{\partial p_t^A} = \frac{N_{t-1}H'(\sigma_{ot})\,u'(c_{ot})}{\left[N_{t-1}H'(\sigma_{ot}) + N_t H'(\sigma_{yt})\right]} \quad (25)$$

$$\frac{\partial\delta}{\partial \tau_t^A} = \frac{N_t H'(\sigma_{yt})\,\beta u'(c_{ot+1})\left[-w(1+R) + \partial p_{t+1}^*/\partial \tau_t^A\right]}{\left[N_{t-1}H'(\sigma_{ot}) + N_t H'(\sigma_{yt})\right]} \quad (26)$$

$$\frac{\partial\delta}{\partial a_{t+1}^A} = \frac{N_t H'(\sigma_{yt})\,\beta u'(c_{ot+1})\,\partial p_{t+1}^*/\partial a_{t+1}^A}{\left[N_{t-1}H'(\sigma_{ot}) + N_t H'(\sigma_{yt})\right]} \quad (27)$$

Conditions 16, 19 and 22 imply that:

$$F'(.)\frac{\partial\delta}{\partial p_t^A}(.) - \lambda_1^{A*} \begin{array}{lll} = 0 & if & p_t^A > 0 \\ \leq 0 & if & p_t^A = 0 \end{array} \quad (28)$$

In turn, equations 16, 19 and 24 imply that:

$$F'(.)\frac{\partial\delta}{\partial a_{t+1}^A}(.) - \lambda_1^{A*}(1+n)^2 \begin{array}{lll} = 0 & if & a_{t+1}^A > 0 \\ \leq 0 & if & a_{t+1}^A = 0 \end{array} \quad (29)$$

Therefore, if $p_t \geq 0$ is not binding, 28 and 29 imply:

$$\frac{\partial \delta}{\partial a_{t+1}^A}(.) - \frac{\partial \delta}{\partial p_t^A}(.)(1+n)^2 \begin{array}{lll} = 0 & if & a_{t+1}^A > 0 \\ \leq 0 & if & a_{t+1}^A = 0 \end{array}$$

and using 25 and 27:

$$H'(\sigma_{yt})\,\beta u'\left(c_{ot+1}^*\right)\frac{\partial p_{t+1}^*}{\partial a_{t+1}^A} - H'(\sigma_{ot})\,u'(c_{ot}^*)(1+n) \begin{array}{lll} = 0 & if & a_{t+1}^A > 0 \\ \leq 0 & if & a_{t+1}^A = 0 \end{array}$$

Using 14 and rearraging:

$$\frac{u'(c_{ot}^*)}{\beta u'\left(c_{ot+1}^*\right)} \left\{ \begin{array}{ll} = \frac{1}{1+n}\partial p_{t+1}^*/\partial a_{t+1}^A & , if \ \ a_{t+1}^A > 0 \\ \geq \frac{1}{1+n}\partial p_{t+1}^*/\partial a_{t+1}^A & , if \ \ a_{t+1}^A = 0 \end{array} \right.$$

provided that $p_t \geq 0$ is not binding. An analogous condition holds true por candidate B, so the superindex A can be dropped at this stage. Also to simplify notation, I drop the stars that indicate equilibrium values, in the understanding that all what follows refers to the equilibrium paths. ■

In the interior solution ($a_{t+1} > 0$), the marginal rate of substitution in the left hand side of 15 equals the marginal rate of transformation in the right hand side of the equation. In a corner solution ($a_{t+1} = 0$), the marginal rate of substitution is equal to or larger than the marginal rate of transformation. In the latter case, politicians would like to increase current pensions reducing the pension fund, to take advantage of the comparatively high marginal utility of old citizens that implies a high responsiveness of old citizens to transfers in terms of votes. But politicians cannot do it, because the pension fund is already at its minimum.

Politicians will not leave a positive pension fund, unless they expect a positive response of the next period pension to the pension fund. Leaving a positive fund involves a sacrifice in terms of current pensions, which means a sacrifice of votes from the old citizens. These votes lost from old citizens must be compensated by votes gained from young citizens who expect larger next period pensions due to a larger pension fund. More formally, $a_{t+1} > 0$ implies that the ratio of marginal utilities in the left hand side of 15 equals $\frac{1}{1+n}\partial p_{t+1}/\partial a_{t+1}$. For this equality to hold $\partial p_{t+1}/\partial a_{t+1} > 0$, since marginal utilities are not negative (strictly positive if there is no satiation). Therefore, $\partial p_{t+1}/\partial a_{t+1} > 0$ is a necessary condition for a positive pension fund to exist in equilibrium.

In deciding the contribution rates, politicians must balance the demand of old voters for higher rates and the likely opposition of young voters to it. Old voters favor as large as possible contribution rates, since they do not pay them because they are already retirees, while their pensions positively depend on current contributions. Young voters face a more complex decision problem, for current contribution rates have two opposite effects on their lifetime income. An increase in the contribution rate reduces their young-age disposable income, but it increases their old-age income, if $\partial p_{t+1}/\partial \tau_t > 0$. Young voters opposition to large contribution rates will be stiffer the smaller is the expected response of next period pensions to current contributions. In an interior optimum, the contribution rate must be such that a marginal increase in this rate raises old citizens votes and reduces young citizens votes by the same amount. In a corner solution with zero tax rate, politicians would be willing to reduce the tax rate even further if they could, for the votes they would gain from young voters outweigh the votes they would lose from old voters. The opposite is true in a corner solution with maximum tax rate. The following proposition characterizes these alternative solutions more formally.

Proposition 2 *The equilibrium path with positive pensions verifies:*

$$\frac{u'(c_{ot})}{\beta u'(c_{ot+1})} \begin{matrix} \leq \\ = \\ \geq \end{matrix} \left[1 + R - \frac{1}{w}\frac{\partial p_{t+1}}{\partial \tau_t}\right] \; if \quad \begin{matrix} \tau_t = 0 \\ 0 < \tau_t < \tau \\ \tau_t = \tau \end{matrix} \tag{30}$$

Proof. Equations 16, 19 and 23 imply that:

$$F'(.)\frac{\partial \delta}{\partial \tau_t^A}(.) + \lambda_1^A w(1+n) \begin{matrix} \leq \lambda_2^A & if & \tau_t^A = 0 \\ = \lambda_2^A & if & \tau_t^A > 0 \end{matrix}$$

But 20 imply that $\lambda_2^A = 0$ if $\tau_t < \tau$, and $\lambda_2^A \geq 0$ if $\tau_t^A = \tau$. Hence:

$$F'(.)\frac{\partial \delta}{\partial \tau_t^i}(.) + \lambda_1^A w(1+n) \begin{matrix} \leq \\ = \\ \geq \end{matrix} \; 0 \; if \quad \begin{matrix} \tau_t^A = 0 \\ 0 < \tau_t^A < \tau \\ \tau_t^A = \tau \end{matrix} \tag{31}$$

Combining 28 and 31, and provided that $p_t^A \geq 0$ is not binding:

$$\frac{\partial \delta}{\partial \tau_t^A}(.) + \frac{\partial \delta}{\partial p_t^A}(.)\, w(1+n) \begin{matrix} \leq \\ = \\ \geq \end{matrix} \; 0 \; if \quad \begin{matrix} \tau_t^A = 0 \\ 0 < \tau_t^A < \tau \\ \tau_t^A = \tau \end{matrix}$$

and using 14, 25 and 26:

$$\beta u'(c_{ot+1})\left[-w(1+R)+\partial p_{t+1}/\partial\tau_t^A\right]+wu'(c_{ot})\begin{array}{c}\leq\\=\\\geq\end{array}0\ if\quad\begin{array}{c}\tau_t^A=0\\0<\tau_t^A<\tau\\\tau_t^A=\tau\end{array}$$

and rearraging:

$$\frac{u'(c_{ot})}{\beta u'(c_{ot+1})}\begin{array}{c}\leq\\=\\\geq\end{array}\left[1+R-\frac{1}{w}\frac{\partial p_{t+1}}{\partial\tau_t^A}\right]\ if\quad\begin{array}{c}\tau_t^A=0\\0<\tau_t^A<\tau\\\tau_t^A=\tau\end{array}$$

In what follows, I drop the superindex to simplify notation, using that candidate B face analogous conditions. ■

The benchmark case in the static model of section 2 in which net transfers are zero corresponds to a fully funded pension system in the dynamic model. In such a system, pensions are: $p_0=0$ and $p_t=w\tau_{t-1}(1+R)\,,\,t>0$, implying that life-time net income is the same for all generations and equal to: $\omega_t=w(1+R)\,,t\geq-1$. Also all generations have the same consumption profile in this system.The question then is whether a fully funded pension system can possibly be a political equilibrium and if so, which are the conditions for this equilibrium to hold. The following proposition deals with this issue.

Proposition 3 *The politico-economic equilibrium embodies a fully funded pension system only if: a)* $\frac{\partial p_{t+1}}{\partial a_{t+1}}(\tau_t,a_{t+1})=\frac{1+n}{\beta}$ *and b)* $\frac{\partial p_{t+1}}{\partial\tau_t}(\tau_t,a_{t+1})\geq w(1+R)-\frac{w}{\beta}$*, for all* $t\geq0$*.*

Proof. In a fully funded pension system the fund is positive and hence, according to proposition 1:

$$\frac{u'(c_{ot})}{\beta u'(c_{ot+1})}=\frac{1}{1+n}\partial p_{t+1}/\partial a_{t+1} \tag{32}$$

Life-time net income is the same for all generations: $\omega_t=w(1+R)$. Since the interest rate was also assumed constant, old-age consumption must be the same for all generations $c_{ot}=c_{ot+1}$. Condition (a) in this proposition then follows from substituting $c_{ot} = c_{ot+1}$ into 32. The contribution rate is positive in a funded system $\tau_t > 0$ and hence, according to proposition 2:

$$\frac{u'(c_{ot})}{\beta u'(c_{ot+1})} \geq 1 + R - \frac{1}{w}\partial p_{t+1}/\partial \tau_t \tag{33}$$

Condition (b) in this proposition follows from substituting $c_{ot} = c_{ot+1}$ into 33 and rearraging terms. ■

This proposition implies that a fully funded system is possible, but only as a limiting case. Next period pensions must be exactly as sensitive to the pension fund as established in condition (a) and "sufficiently" sensitive to current taxes as established in condition (b) for the fully funded scheme to be sustainable as part of a politico-economic equilibrium.The pension system cannot be fully funded if condition (a) does not hold, but it can still be partially funded. The following proposition shows that young citizens are worse off than old cotizens in a partially funded system, if pensions are expected to be less sensitive to the fund than what it is needed to support a fully funded pension scheme.

Proposition 4 *If the equilibrium pension fund is positive in* $t+1$*, but* $\frac{\partial p_{t+1}}{\partial a_{t+1}}(\tau_t, a_{t+1}) < \frac{1+n}{\beta}$*, then* $c_{ot+1} < c_{ot}$*.*

Proof. Proposition 1 says that candidates in period-t elections will not leave a positive fund unless $(1+n)\, u'(c_{ot}) / \beta u'(c_{ot+1}) = \partial p_{t+1}/\partial a_{t+1}$. Therefore, if $a_{t+1} > 0$ and $\frac{\partial p_{t+1}}{\partial a_{t+1}}(\tau_t, a_{t+1}) < \frac{1+n}{\beta}$, then $u'(c_{ot}) / u'(c_{ot+1}) < 1$ and $c_{ot+1} < c_{ot}$. ■

5 A parametric example

Suppose utility is logarithmic: $u(c) = Ln(c)$. Then, optimal old-age consumption is:

$$c_{ot+1} = \frac{\beta}{1+\beta}\omega_t \tag{34}$$

and equations 15 and 30 can be rewritten as:

$$\frac{u'(c_{ot})}{\beta u'(c_{ot+1})} = \frac{\omega_t}{\beta \omega_{t-1}} \begin{cases} = \frac{1}{1+n}\partial p_{t+1}/\partial a_{t+1} & , if \quad a_{t+1} > 0 \\ \geq \frac{1}{1+n}\partial p_{t+1}/\partial a_{t+1} & , if \quad a_{t+1} = 0 \end{cases} \tag{35}$$

$$\frac{u'(c_{ot})}{\beta u'(c_{ot+1})} = \frac{\omega_t}{\beta \omega_{t-1}} \begin{matrix} \leq \\ = \\ \geq \end{matrix} \left[1 + R - \frac{1}{w}\frac{\partial p_{t+1}}{\partial \tau_t}\right] \; if \quad \begin{matrix} \tau_t = 0 \\ 0 < \tau_t < \tau \\ \tau_t = \tau \end{matrix} \tag{36}$$

The wealth of generation t is:

$$\omega_t = w(1-\tau_t)(1+R) + p_{t+1} \;\;,\;\; t \geq -1 \tag{37}$$

implying that:

$$\frac{\partial p_{t+1}}{\partial a_{t+1}} = \frac{\partial \omega_t}{\partial a_{t+1}} \tag{38}$$

$$\frac{\partial p_{t+1}}{\partial \tau_t} = \frac{\partial \omega_t}{\partial \tau_t} + w(1+R) \tag{39}$$

and using 38 and 39 in 35 and 36:

$$\frac{\omega_t}{\beta \omega_{t-1}} \left\{ \begin{matrix} = \frac{1}{1+n}\partial\omega_t/\partial a_{t+1} & , \; if \quad a_{t+1} > 0 \\ \geq \frac{1}{1+n}\partial\omega_t/\partial a_{t+1} & , \; if \quad a_{t+1} = 0 \end{matrix} \right. \tag{40}$$

$$\frac{\omega_t}{\beta \omega_{t-1}} \begin{matrix} \leq \\ = \\ \geq \end{matrix} -\frac{1}{w}\partial\omega_t/\partial\tau_t \; if \quad \begin{matrix} \tau_t = 0 \\ 0 < \tau_t < \tau \\ \tau_t = \tau \end{matrix} \tag{41}$$

It will prove useful to write the budget constraint of the government in terms of the individuals life-time income. Using 37 to substitute for pensions in 3:

$$\begin{aligned} \omega_{t-1} &= a_t(1+n)(1+R) + [w(1+R)(1-\tau_{t-1}) + w(1+n)\tau_t] - a_{t+1}(1+n)^2 \\ &= a_t(1+n)(1+R) + z_{t-1} - a_{t+1}(1+n)^2 \;\;,\;\; t \geq 0 \end{aligned} \tag{42}$$

where z_{t-1} is the wealth of generation $t-1$ when the pension system is PAYG: $z_{t-1} = w(1+R)(1-\tau_{t-1}) + (1+n)\tau_t$. Equations 40, 41 and 42 conform a system of first order difference equations in ω_t, a_t, and τ_t.

The main difficulty to characterize the solution is to determine the unknown function $\omega_t(a_{t+1}, \tau_t)$. The following two propositions do it. Later on, using these results, I characterize conditions such that there is a fully funded pension system in equilibrium (proposition 7), there is no pension system in equilibrium (proposition 9) and the pension system is unfunded or at most partially funded in the transition equilibrium path and becomes PAYG in the steady state (proposition 10).

Proposition 5 says that if the system is in a steady state and politicians find it optimal to exhaust the pension fund, they must also find it optimal to charge as large as possible a contribution rate. If the non-negativity constraint on the pension fund is binding, politicians want to reduce the pension fund further to offer better benefits to old citizens. But they are constrained by their inability to borrow. For this to be optimal, politicians must also be constrained in the contribution rate. Otherwise they could still gain votes benefitting the still highly responsive old voters raising contributions paid by young voters.

Proposition 5 *In a steady state in which $p_t \geq 0$ is not binding, $a_{t+1} \geq 0$ is binding if and only if $\tau_t \leq \tau$ is binding.*

Proof. Suppose the economy is in a steady state from period T onwards, i.e. $(a_{t+1}, \tau_t, \omega_t) = (a_{SS}, \tau_{SS}, \omega_{SS})$, $t \geq T$. Then 42 implies that

$$\omega_{SS} = a_{SS} (1+n)(R-n) + w(1+R) - w\tau_{SS}(R-n)$$

and taking derivatives: $\partial\omega_{SS}/\partial a_{SS} = (1+n)(R-n)$ and $\partial\omega_{SS}/\partial\tau_{SS} = -w(R-n)$. But this computations mean that the right hand sides in 40 and 41 are the same. Therefore, if $p_t \geq 0$ is not binding, then:

$$\frac{\omega_t}{\beta\omega_{t-1}} > \frac{1}{1+n}\partial\omega_t/\partial a_{t+1} = R-n \iff \frac{\omega_t}{\beta\omega_{t-1}} > -\frac{1}{w}\partial\omega_t/\partial\tau_t = R-n$$

and $a_{t+1} \geq 0$ is binding if and only if $\tau_t \leq \tau$ is also binding, for $t \geq T$. ■

Proposition 6 *Suppose there is a politico-economic equilibrium in wich the constraint that pensions are not negative is not binding. Then, the wealth of citizens in this equilibrium is given by the following expressions:*

$$\omega_t = a_{t+1}(1+n)(1+R) + w(1+R)(1-\tau_t) + w(1+n)\tau, \quad (43)$$
$$if \ a_{t+2} = 0$$

$$
\begin{aligned}
\omega_t &= \frac{a_{t+1}(1+n)(1+R) - w(1+R)\tau_t + x}{1+\beta(1+n)}, \\
x &= w(1+R)\sum_{q=0}^{Q}\left(\frac{1+n}{1+R}\right)^q + w(1+n)\tau\left(\frac{1+n}{1+R}\right)^Q \quad , \\
&if\ \ a_{t+Q+2}=0,\ a_{t+Q+1}>0,...,a_{t+2}>0
\end{aligned} \tag{44}
$$

Proof. Suppose first that in $T-1$ the system reaches a steady state in which $p_{T-1} \geq 0$ is not binding and the pension fund is zero, i.e. $a_T = a_{T+1} = ... = 0$. By virtue of the previous proposition, $\tau_{T-1} = \tau_T = ... = \tau$. The functions $\omega_{T-1}(a_T, \tau_{T-1})$ and $\omega_{T-2}(a_{T-1}, \tau_{T-2})$ can then be computed from 42 as:

$$
\omega_{T-1} = w(1+R)(1-\tau) + w(1+n)\tau, \quad if\ \ a_T = a_{T+1} = ... = 0
$$

$$
\begin{aligned}
\omega_{T-2} &= a_{T-1}(1+n)(1+R) + w(1+R)(1-\tau_{T-2}) + w(1+n)\tau \\
&= a_{T-1}(1+n)(1+R) + z_{T-2}, \qquad if\ \ a_T = a_{T+1} = ... = 0
\end{aligned} \tag{45}
$$

The life-time income of the generation born in $T-3$ can now be computed using 40 and 45:

$$
\frac{\omega_{T-2}}{\beta\omega_{T-3}} \begin{cases} = \frac{1}{1+n}\partial\omega_{T-2}/\partial a_{T-1} & , if\ \ a_{T-1}>0 \\ \geq \frac{1}{1+n}\partial\omega_{T-2}/\partial a_{T-1} & , if\ \ a_{T-1}=0 \end{cases}
$$

Equation 45 implies that $\partial\omega_{T-2}/\partial a_{T-1} = (1+n)(1+R)$, and hence:

$$
\frac{\omega_{T-2}}{\beta\omega_{T-3}} \begin{cases} = 1+R & , if\ \ a_{T-1}>0 \\ \geq 1+R & , if\ \ a_{T-1}=0 \end{cases} \tag{46}
$$

If $a_{T-1} = 0$, the same argument used to compute generation $T-2$ life-time income applies for generation $T-3$:

$$
\begin{aligned}
\omega_{T-3} &= a_{T-2}(1+n)(1+R) + z_{T-3} \\
z_{T-3} &= w(1+R)(1-\tau_{T-3}) + w(1+n)\tau, \quad if\ \ a_{T-1}=0
\end{aligned} \tag{47}
$$

If $a_{T-1} > 0$, from 45 and 46:

$$
\omega_{T-3} = \frac{\omega_{T-2}}{\beta(1+R)} = \frac{a_{T-1}(1+n)(1+R) + z_{T-2}}{\beta(1+R)}, \quad if\ \ a_{T-1}>0,\ a_T=0 \tag{48}
$$

From 42:

$$\begin{aligned} a_{T-1}(1+n)^2 &= a_{T-2}(1+n)(1+R) + z_{T-3} - \omega_{T-3} \\ z_{T-3} &= (1+R)\,w\,(1-\tau_{T-3}) + (1+n)\,w\tau_{T-2} \end{aligned}$$

in 48:

$$\omega_{T-3} = \frac{a_{T-2}(1+n)(1+R) + z_{T-3} + z_{T-2}\left(\frac{1+n}{1+R}\right)}{1+\beta(1+n)}, \quad if \;\; a_{T-1} > 0,\, a_T = 0 \tag{49}$$

and the function $\omega_{T-3}(a_{T-2}, \tau_{T-3})$ follows from substituting z_{T-3} and z_{T-2} in 49:

$$\begin{aligned} \omega_{T-3} &= \frac{a_{T-2}(1+n)(1+R) + w(1+R)(1-\tau_{T-3}) + w(1+n)\left[1+\tau\left(\frac{1+n}{1+R}\right)\right]}{1+\beta(1+n)}, \\ if \;\; a_{T-1} &> 0,\, a_T = 0 \end{aligned} \tag{50}$$

I turn now to the life-time income of the generation born in $T-4$. From 42:

$$\omega_{T-4} = a_{T-3}(1+n)(1+R) + z_{T-4} - a_{T-2}(1+n)^2 \tag{51}$$

From 40:

$$\frac{\omega_{T-3}}{\beta\omega_{T-4}} \begin{cases} = \frac{1}{1+n}\partial\omega_{T-3}/\partial a_{T-2} & , \; if \;\; a_{T-2} > 0 \\ \geq \frac{1}{1+n}\partial\omega_{T-3}/\partial a_{T-2} & , \; if \;\; a_{T-2} = 0 \end{cases} \tag{52}$$

and from 47 and 50:

$$\frac{\partial\omega_{T-3}}{\partial a_{T-2}} \begin{cases} = (1+n)(1+R) & , \; if \;\; a_{T-1} = 0 \\ = \frac{(1+n)(1+R)}{1+\beta(1+n)} & , \; if \;\; a_{T-1} > 0 \end{cases} \tag{53}$$

There are four paths to consider: a) $a_{T-2} = a_{T-1} = a_T = 0$, b) $a_{T-2} > 0, a_{T-1} = a_T = 0$, c) $a_{T-2} = 0, a_{T-1} > 0,\, a_T = 0$, d) $a_{T-2} > 0, a_{T-1} > 0,\, a_T = 0$.

a) $a_{T-2} = a_{T-1} = a_T = 0$. From 51:

$$\omega_{T-4} = a_{T-3}(1+n)(1+R) + z_{T-4}, \; if \; a_{T-2} = 0 \tag{54}$$

In this path, the economy is already in a steady state in period $T-4$ and, by virtue of the previous proposition, $a_{T-2} = 0$ implies that $\tau_{T-3} = \tau$. Therefore:

$$\begin{aligned} \omega_{T-4} &= a_{T-3}(1+n)(1+R) + w(1+R)(1-\tau_{T-4}) + w(1+n)\tau, \\ if \; a_{T-2} &= a_{T-1} = a_T = 0 \end{aligned} \tag{55}$$

b) $a_{T-2} > 0, a_{T-1} = a_T = 0$. Equations 52 and 53 imply that:

$$\omega_{T-4} = \frac{\omega_{T-3}}{\beta(1+R)}, \quad if\ a_{T-2} > 0, a_{T-1} = a_T = 0 \tag{56}$$

In turn, 47 and 56 imply:

$$\omega_{T-4} = \frac{a_{T-2}(1+n)(1+R) + z_{T-3}}{\beta(1+R)}, \quad if \quad a_{T-2} > 0, a_{T-1} = a_T = 0 \tag{57}$$

From 42:

$$a_{T-2}(1+n)^2 = a_{T-3}(1+n)(1+R) + z_{T-4} - \omega_{T-4} \tag{58}$$

Substituting in 57:

$$\omega_{T-4} = \frac{a_{T-3}(1+n)(1+R) + z_{T-4} + z_{T-3}\left(\frac{1+n}{1+R}\right)}{1+\beta(1+n)}, \quad if \quad a_{T-2} > 0, a_{T-1} = a_T = 0 \tag{59}$$

and the function $\omega_{T-4}(a_{T-3}, \tau_{T-4})$ follows from substituting z_{T-4} and z_{T-3} in 59:

$$\omega_{T-4} = \frac{a_{T-3}(1+n)(1+R) + w(1+R)(1-\tau_{T-4}) + w(1+n)\left[1+\tau\left(\frac{1+n}{1+R}\right)\right]}{1+\beta(1+n)},$$
$$if \ \ a_{T-2} > 0, a_{T-1} = a_T = 0 \tag{60}$$

c) $a_{T-2} = 0,\ a_{T-1} > 0,\ a_T = 0$. From 50:

$$\frac{\partial \omega_{T-3}}{\partial a_{T-2}} = \frac{(1+n)(1+R)}{1+\beta(1+n)}$$
$$\frac{\partial \omega_{T-3}}{\partial \tau_{T-3}} = -\frac{w(1+R)}{1+\beta(1+n)}$$

But this computations mean that the right hand sides in 40 and 41 are the same. Therefore, if $p_{T-3} \geq 0$ is not binding, then:

$$\frac{\omega_{T-3}}{\beta\omega_{T-4}} > \frac{1}{1+n}\frac{\partial \omega_{T-3}}{\partial a_{T-2}} = \frac{(1+R)}{1+\beta(1+n)} \iff \frac{\omega_{T-3}}{\beta\omega_{T-4}} > -\frac{1}{w}\frac{\partial \omega_{T-3}}{\partial \tau_{T-3}} = \frac{(1+R)}{1+\beta(1+n)}$$

and $a_{T-2} \geq 0$ is binding if and only if $\tau_{T-3} \leq \tau$ is also binding. Using this result in 54:

$$\begin{aligned} \omega_{T-4} &= a_{T-3}(1+n)(1+R) + w(1+R)(1-\tau_{T-4}) + w(1+n)\tau, \\ if\ a_{T-2} &= 0,\ a_{T-1} > 0,\ a_T = 0 \end{aligned} \tag{61}$$

d) $a_{T-2} > 0, a_{T-1} > 0,\ a_T = 0$. Equations 52 and 53 imply that:

$$\omega_{T-4} = \frac{1+\beta(1+n)}{\beta(1+R)}\omega_{T-3}, \quad if\ a_{T-2} > 0,\ a_{T-1} > 0,\ a_T = 0 \tag{62}$$

Life-time income of generation born in $T-3$ is given by 49 along this path, and hence substituting back into 62:

$$\begin{aligned} \omega_{T-4} &= \frac{a_{T-2}(1+n)(1+R) + z_{T-3} + z_{T-2}\left(\frac{1+n}{1+R}\right)}{\beta(1+R)}, \\ if\ a_{T-2} &> 0,\ a_{T-1} > 0,\ a_T = 0 \end{aligned} \tag{63}$$

and substituting a_{T-2} from 58 into 63:

$$\begin{aligned} \omega_{T-4} &= \frac{a_{T-3}(1+n)(1+R) + z_{T-4} + z_{T-3}\left(\frac{1+n}{1+R}\right) + z_{T-2}\left(\frac{1+n}{1+R}\right)^2}{1+\beta(1+n)}, \\ if\ a_{T-2} &> 0,\ a_{T-1} > 0,\ a_T = 0 \end{aligned}$$

Repeating the argument, the following formulae emerge for life-time income in equilibrium, when the fund is expected to become zero in period T:

$$\begin{aligned} \omega_{T-2} &= a_{T-1}(1+n)(1+R) + z_{T-2}, \\ z_{T-2} &= w(1+R)(1-\tau_{T-2}) + w(1+n)\tau \quad if \quad a_T = 0 \end{aligned} \tag{64}$$

and

$$\begin{aligned} \omega_{T-2-Q} &= \frac{a_{T-1-Q}(1+n)(1+R) + \sum_{q=0}^{Q} z_{T-2-Q+q}\left(\frac{1+n}{1+R}\right)^q}{1+\beta(1+n)}, \\ z_{T-2} &= w(1+R)(1-\tau_{T-2}) + w(1+n)\tau, \\ if\ a_{T-Q} &> 0, ..., a_{T-1} > 0,\ a_T = 0, Q \geq 1 \end{aligned} \tag{65}$$

Equation 43 follows after relabeling the time periods as $t = T-2$ in 64:

$$\begin{aligned} \omega_t &= a_{t+1}(1+n)(1+R) + z_t, \\ z_t &= w(1+R)(1-\tau_t) + w(1+n)\tau \quad if \quad a_{t+2} = 0 \end{aligned} \tag{66}$$

Relabeling $t = T - 2 - Q$ in 65:

$$\begin{aligned} \omega_t &= \frac{a_{t+1}(1+n)(1+R) + \sum_{q=0}^{Q} z_{t+q}\left(\frac{1+n}{1+R}\right)^q}{1+\beta(1+n)}, \quad (67) \\ z_{t+Q} &= w(1+R)(1-\tau_{t+Q}) + w(1+n)\tau, \\ if\ a_{t+2} &> 0, ..., a_{t+Q+1} > 0,\ a_{t+Q+2} = 0, Q \geq 1 \end{aligned}$$

Finally, notice that:

$$\begin{aligned} &\sum_{q=0}^{Q} z_{t+q}\left(\frac{1+n}{1+R}\right)^q = \\ &w(1+R)\sum_{q=0}^{Q}\left(\frac{1+n}{1+R}\right)^q + w(1+n)\tau\left(\frac{1+n}{1+R}\right)^Q - w(1+R)\tau_t, \\ &if\quad a_{t+Q+2} = 0 \end{aligned}$$

which substituted into 67 yields 44. ■

A fully funded pension system could be part of a politico-economic equilibrium, but if and only if the exogenous "time rates" of the economy happen to satisfy a precise condition. According to the following proposition, the rate of time preference, the rate of interest and the rate of growth of the population must be such that $\beta(R-n) = 1$. This is a necessary and sufficient condition for the wealth of all generations to be equal to $w(1+R)$ in equilibrium, which means that the pension system causes no redistribution in these equilibria, i.e. it is fully funded. Citizens and political candidates are indifferent as to the size of the pension program when this condition holds, implying that the model exhibits a continuum of equilibria with $0 \leq \tau_t \leq \tau$ and $p_{t+1} = w(1+R)\tau_t$. This indeterminacy is a natural result in a model in which the only role of pensions is to perform redistribution.

Proposition 7 *The pension system is fully funded in a politico-economic equilibrium if and only if $\beta(R-n) = 1$. The size of the pension program is indeterminate, in the sense that any feasible τ_t is consistent with these equilibria.*

Proof. Necessity (only if): In a fully funded program, $a_{t+1} > 0$, for all $t \geq 0$. For this to be part of an equilibrium in which pensions are positive (or, more precisely, the non-negativity constraints on pensions are not binding), condition 44 with $Q \to \infty$ must hold for generations $t \geq -1$. Using 44 to compute $\partial \omega_t / \partial a_{t+1}$ in 40:

$$\frac{\omega_t}{\beta \omega_{t-1}} = \frac{(1+R)}{1+\beta(1+n)} , \quad t \geq 0 \tag{68}$$

Also, in a fully funded system, $\omega_t = w(1+R)$, and hence $\omega_t / \omega_{t-1} = 1$, for all t. This implies that

$$\frac{1}{\beta} = \frac{(1+R)}{1+\beta(1+n)} , \quad t \geq 0 \tag{69}$$

and therefore $\beta(R-n) = 1$, from $t = 0$ onwards.
Sufficiency (if): Equations 43 and 44 in 40 imply that $\omega_t \geq \omega_{t-1}$, $t \geq 0$, if $\beta(R-n) = 1$. In turn, the first generation can only win with the pension system: $\omega_{-1} = w(1+R) + p_0 \geq w(1+R)$. Therefore, in equilibrium: $\omega_t \geq w(1+R)$, $t \geq -1$, if $\beta(R-n) = 1$. This is to say that all generations must get at least as much as in a fully funded system in the politico-economic equilibrium. Also $\beta(R-n) = 1$ imply that $R > n$ and therefore the intertemporal budget constraint 6 holds. The only way this constraint is honored with $\omega_t \geq w(1+R)$, $t \geq -1$ is that each and all generations get exactly $\omega_t = w(1+R)$, $t \geq -1$. This is the individuals wealth that is associated to a fully funded pension program, but it is also the wealth associated to the absence of pensions, so it remains to be proved that there are equilibria in which $\tau_t > 0$ and $p_{t+1} > 0$. Notice first that $a_{t+1} \geq 0$ is not binding in equilibrium if $\beta(R-n) = 1$. The pension fund $a_{T+1}, T \geq 0$, can be computed integrating the per-period budget constraints of the pension system as follows:

$$\begin{aligned} & \sum_{t=0}^{T} \left[w(1+R) - \omega_{t-1} \right] \left(\frac{1+n}{1+R} \right)^t + w(1+n)\tau_T \left(\frac{1+n}{1+R} \right)^T \\ = \; & a_{T+1}(1+n)^2 \left(\frac{1+n}{1+R} \right)^T \end{aligned}$$

Using that $\omega_{t-1} = w(1+R)$, it follows that $a_{T+1} = w\tau_T / (1+n)$, i.e. political candidates want to have the pension fund proportional to the current contribution rate in these equilibria. Therefore, $a_{T+1} \geq 0$ imposes no additional constraint on top of $\tau_T \geq 0$. Thus, using 44 to compute $\partial \omega_t / \partial \tau_t$:

$$\partial \omega_t / \partial \tau_t = \frac{-w(1+R)}{1+\beta(1+n)} , \quad t \geq 0$$

Substituting it in 36:

$$\frac{\omega_t}{\beta\omega_{t-1}} \begin{matrix} \leq \\ = \\ \geq \end{matrix} \frac{(1+R)}{1+\beta(1+n)} \; if \; \begin{matrix} \tau_t = 0 \\ 0 < \tau_t < \tau \\ \tau_t = \tau \end{matrix} \tag{70}$$

But we already know that $\omega_t = \omega_{t-1}$ and that $\beta(1+R)/[1+\beta(1+n)] = 1$, if $\beta(R-n) = 1$. Therefore, the left and the right hand sides are equal in 70, implying that any τ_t such that $(0 \leq \tau_t \leq \tau)$ is consistent with an equilibrium. ■

Corollary 8 *A fully funded pension program cannot be sustained in a politico-economic equilibrium in a dynamically inefficient economy.*

Proof. Proposition 7 says that $\beta(R-n) = 1$ is a necessary condition for a fully funded pension program to be part of a politico-economic equilibrium. But this condition does not hold in a dynamically inefficient economy, for $n > R$ in this case. ■

The following proposition shows that there is no pension program in equilibrium if $\beta(R-n) > 1$. An equilibrium with positive pensions would require an ever increasing life-time income of successive generations if $\beta(R-n) > 1$, and this is not consistent with the intertemporal budget constraint of the social security system.

Proposition 9 *There is no pension program in equilibrium if* $\beta(R-n) > 1$.

Proof. I show in part (i) of this proof that, with $\beta(R-n) > 1$, the non negativity constraints $p_t \geq 0$ and $\tau_t \geq 0$ cannot be binding in any $t \geq 1$, if they are not binding in period 0. Then, in part (ii), I show that an equilibrium path in which $p_t \geq 0$ is never binding is not consistent with the social security intertemporal budget constraint, if $\beta(R-n) > 1$. Therefore, the assumption that $p_0 \geq 0$ and $\tau_0 \geq 0$ are not binding drives to a contradiction, and the only remaining possibility is that $p_t = \tau_t = 0$, for all $t \geq 0$. (i) Suppose, that $p_0 \geq 0$ and $\tau_0 \geq 0$ are not binding. I first show that $p_1 \geq 0$ cannot be binding either. Indeed, according to 36, $\tau_0 \geq 0$ is not binding only if $\omega_0/\beta\omega_{-1} \geq 1+R-(1/w)\,\partial p_1/\partial\tau_0$. If $p_1 \geq 0$ were to be binding, then $\partial p_1/\partial\tau_0 = 0$, and the necessary condition for a non-binding non-negativity pension contribution in 0 would become $\omega_0/\beta\omega_{-1} \geq 1+R$. But this would mean that $\omega_0 > \omega_{-1}$, if $\beta(R-n) > 1$. Also $\omega_0 - \omega_{-1} = w(1+R)(1-\tau_0) + p_1 - w(1+R) - p_0 > 0$

and then $p_1 - p_0 > w(1+R)\tau_0 \geq 0$, which means that $p_1 \geq 0$ is not binding if $p_0 \geq 0$ and $\tau_0 \geq 0$ are not binding. Now, $\omega_0 > \omega_{-1}$ implies that $\tau_1 > 0$, for $\omega_0 > \omega_{-1} \geq w(1+R)$ implies that $p_1 - w(1+R)\tau_0 > 0$ which is only possible if $\tau_1 > 0$. Indeed, from the budget constraint of the pension system:

$$p_1 - w(1+R)\tau_0 = w(1+n)\tau_1 - p_0\left(\frac{1+R}{1+n}\right) - a_2(1+n)^2 \tag{71}$$

and since $p_0 \geq 0$, $a_2 \geq 0$, the only way the left side be positive is that $\tau_1 > 0$. A similar argument can be used to show that $p_2 \geq 0$ is not binding and $\tau_2 > 0$ along this path. According to 36, $\tau_1 > 0$ in equilibrium only if $\omega_1/\beta\omega_0 \geq 1 + R - (1/w)\,\partial p_2/\partial\tau_1$. If $p_2 \geq 0$ were to be binding, then $\partial p_2/\partial\tau_1 = 0$, and $\omega_1/\beta\omega_0 \geq 1+R$. But this means that $\omega_1 > \omega_0 > \omega_{-1}$, if $\beta(R-n) > 1$. Also $\omega_1 - \omega_{-1} = w(1+R)(1-\tau_1) + p_2 - w(1+R) - p_0 > 0$ and then $p_2 - p_0 > w(1+R)\tau_1 \geq 0$, which means that $p_2 \geq 0$ would not be binding if $p_0 \geq 0$ and $\tau_1 \geq 0$ were not binding. Now, $\omega_1 > \omega_0$ implies that $\tau_2 > 0$, for $\omega_1 > \omega_0 > \omega_{-1} \geq w(1+R)$ implies that $p_2 - w(1+R)\tau_1 > 0$ which is only possible if $\tau_2 > 0$. Exactly the same argument applies for the following periods and therefore, there is no period in which $p_t \geq 0$ and $\tau_t \geq 0$ are binding if $p_0 \geq 0$ and $\tau_0 \geq 0$ are not binding. (ii) Notice that along this path $\omega_{-1} \geq w(1+R)$ and $\omega_t > w(1+R)$, $t \geq 0$, if $\beta(R-n) > 1$ and $p_t \geq 0$ is not binding. But this is not consistent with the intertemporal budget constraint 6. Therefore, $p_t = \tau_t = 0$, $t \geq 0$ in equilibrium, if $\beta(R-n) > 1$.
■

The equilibrium path exhibits a pay-as-you-go pension program in the long run, if $\beta(R-n) < 1$. In this steady state, the contribution rate must be at its maximum. Politicians would be willing to reduce the pension fund and to raise the contribution rate even further to pay more generous pensions, but they cannot because the pension fund has reached its lower bound and the contribution rate has reached its upper bound.

Proposition 10 *There is no pension fund in the steady state if $\beta(R-n) < 1$. The contribution rate is at its maximum in this steady state.*

Proof. Suppose, to the contrary, that there is a positive pension fund in equilibrium and that this fund continues positive for ever. Then, from 40 and 44:

$$\frac{\omega_t}{\omega_{t-1}} = \frac{\beta(1+R)}{1+\beta(1+n)} < 1\,, \quad t \geq 0 \tag{72}$$

and hence the life-time income of successive generations would be decreasing, and this could not be a steady state. Therefore, $a_{t+1} \geq 0$ is binding in the steady state. Proposition 5 then implies that $\tau_t \leq \tau$ is also binding in the steady state. ■

Proposition 11 *The welfare effects of political competition through the pension system can be summarized as follows: a) no generation is affected if* $\beta(R-n) \geq 1$*; b) the first generations win at the expense of all the succeeding ones if* $0 < \beta(R-n) < 1$*; c) all generations win if* $\beta(R-n) \leq 0$.

Proof. a) According to proposition 9, there is no pension program in equilibrium if $\beta(R-n) > 1$, and, according to proposition 7, there is a fully funded pension program if $\beta(R-n) = 1$. In both cases, the life-time income and welfare of all generations remain untouched. b) If $\beta(R-n) < 1$, the steady state is characterized by $\tau_{SS} = \tau$ and $\alpha_{SS} = 0$ (proposition 10) and the life-time income of the generations that are born when the system is in the steady state is $\omega_{SS} = w(1+R)(1-\tau) + w(1+n)\tau = w(1+R) + w(n-R)\tau$. If $n < R$, then $\omega_{SS} < w(1+R)$ and these generations lose with the pension system. In turn, the generation whose members are already old when the pension system begins cannot lose, for they pay no contributions when they are active while they may receive pensions when they are old.and hence $\omega_{-1} \geq w(1+R)$. Intermediate generations may gain or lose, depending on parameter values. If the equilibrium path exhibits a transition dynamics with positive pension fund, then equations 40 and 44 imply that the welfare of each generation who is born in this period is smaller than the welfare of the previous generation according to the following formula:

$$\frac{\omega_t}{\omega_{t-1}} = \frac{\beta(1+R)}{1+\beta(1+n)} < 1 \tag{73}$$

c) If $\beta(R-n) \leq 0$, then $\omega_{SS} > w(1+R)$. If the equilibrium path exhibits a transition dynamics with positive pension fund, then generations who are born in this period get no less than ω_{SS}, since equation 73 holds. ■

6 Concluding remarks

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