Tolerance of Noncompliance: Discretion Rather than Simple Rules? 1

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Abstract

We present a simple model of a benevolent government that provides insurance to risk averse individuals on a discretionary basis. As in macroeconomics, commitment to fully contingent rules is better than discretion, but when the government can only commit to simple rules, discretion may be the best available option. The model provides a simple albeit precise characterization of discretion and commitment to a simple rule in the context of social insurance, showing when and why discretion may be better than commitment. We argue that the forces highlighted in our model can provide a rationale for several highly distortive policies often observed in the real world in weak institutional environments, including poor enforcement of norms and soft budgeting.

JEL: E61, H20, H30, H50, O17

Keywords: Discretion, Commitment, Simple Rules, Informality, Enforcement.

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Introduction

In developing countries, economic and social policies often seem to be conducted on a discretionary basis. Governments seem to be unable or unwilling to tie their hands following well established formal rules. Also, there tends to be a big gap between *de jure* and *de facto* policies (Van de Wall, 2001). In this paper, we try to formally explain these stylized facts and provide a characterization of the discretion-commitment dilemma that a benevolent government aimed at maximizing citizens’ welfare face when neither private firms nor the government itself can provide full protection on a pre-committed basis.

Following the seminal papers of Kydland and Prescott (1977) and Calvo (1978) the macroeconomic policy literature clarified the pros and cons of commitment and discretion in monetary and fiscal policies (see Persson and Tabellini 1990, 2000, for surveys). We do not see a parallel development in the social policy literature, even though the dilemma between discretion and commitment is at least as pressing in this field as in macroeconomics. Lindbeck and Weibull (1988) is a remarkable exception. They make the case that the formal welfare states can provide a commitment technology, reducing the distortion caused by discretionary social protection policies.

In the present paper, we borrow ideas from the macroeconomic literature to build a model to think formally about the commitment-discretion dilemma in social insurance (broadly defined). A key ingredient in our story is that in weak institutional environments governments are able to commit only to simple rules, which means incomplete insurance. Simple rules provide the right incentives, but at the cost of too much risk. Discretion provides flexibility, at the cost of large distortions. There is thus a non-trivial tradeoff and, in some cases, discretion may be the right choice.

In the macroeconomic literature, simple rules are defined in opposition to fully contingent rules. The latter are highly sophisticated policy rules that map from each and every state of nature to policy actions. A simple rule is a non-contingent policy. Committing to a fully contingent policy involves no loss of flexibility, as compared to discretion, but committing to a simple rule comes at the cost of some rigidity: the same policy action will be adopted irrespective of the state of nature. In the story the present paper tells, governments are able to commit to only partially contingent rules. Policies are fully contingent under discretion, but pre-committed policy rules involve some loss of flexibility.
The inability of governments to commit to fully contingent rules can be justified on different grounds. One possible reason is that governments may not know the probability of occurrence of some outcomes. If this is the case, it may not be possible to set formal insurance programs. Governments can still act ex-post on a discretionary basis, but they may not be able to design and budget a formal program in this environment with Knightian uncertainty (Knight, 1921). A complementary reason is that some outcomes of individuals’ actions may not be verifiable by a court, even if they are observable by everybody. If this is the case, formal social protection cannot be based on those outcomes and pre-committed rules will not be fully contingent. If outcomes are observable, governments are still able to act ex-post on a discretionary basis. In this interpretation, we borrow from the literature of economics of information and contracts (for surveys of this literature, see among others, Macho-Stadler and Pérez-Castrillo, 1997; Bolton and Dewatripont, 2005; and Salanić, 2005).

This paper is related to several strands of the literature, besides the already mentioned macroeconomic time consistency literature. Our model can be seen as a contribution to the literature about the soft budget constraint. In the discretionary regime in our model, ex-post government interventions to help individuals who faced a negative shock relax individuals’ budget constraint. These interventions protect individuals, but they also distort incentives. The present paper provides a simple theory of why governments –which play the role of the supporting organization–, do not “simply erect an insuperable bureaucratic barrier”, using the words of Kornai et. al. (2003), to get free of the soft budget syndrome. In terms of our model, doing that would mean choosing a simple rule that may provide too little protection.

In many policy areas, discretion often seems to adopt the form of limited enforcement of laws. One simple form of getting the desired “flexibility” is to selectively turn a blind eye on noncompliance, i.e. to choose not to fully enforce the norms in some circumstances. In this light, our analysis of discretionary policies is related to the literature on optimal enforcement initiated with the seminal paper of Becker (1968). However, our motivation and approach are very different. Unlike that literature, we do not analyze noncompliance as an offense to be punished and deterred. On the contrary, in our setting noncompliance is

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4 See however Barr (2001, p 24) for a different view. Making the case of social over private insurance, he argues that, unlike private insurance companies, public agencies can provide insurance even when there is true uncertainty.
something tolerated by the government to help individuals cope with negative shocks. In this sense, while most of the literature focuses on governments’ limited ability to enforce the law, our model provides a theory of why governments might be unwilling to enforce the law.

The topic we are interested in is also related to a large literature on informality. Perry et. al. (2007) identify two types of explanations of informality that emphasize, respectively, exit and exclusion. Conventional cost-benefit analysis is at the core of the exit approach to informality. Firms and workers decide to exit from the formal sector when they consider that the costs outweigh the benefits associated to formality. The second approach views informality mainly as an exclusion phenomenon by which some workers and small firms cannot participate in the formal sector because of excessive or inappropriate regulation.

Limited government monitoring and enforcement capacities are usually important ingredients in the theories of informality. This can explain not only why informality is more prevalent in weak institutional environments but also why tax collecting agencies usually pay more attention to big contributors. With fixed monitoring and collecting costs, it may be optimal to focus on big firms, paying little attention to small firms and the self-employed (Bigio and Zilberman, 2011; Busso et. al. 2012; Bosch, et. al. 2013).

The idea that low government monitoring and enforcement capacity explains the prevalence of informality in developing countries is simple and appealing. Nevertheless, we think this is not the whole story. When it comes, for example, to compliance with the labor code or health and labor security regulations, this explanation of government inaction seems insufficient. In many countries it is well known that large segments of the population work in the informal sector under highly unhealthy and risky conditions. In many middle income countries, we do not find it very convincing the idea that states are so weak that, for example, they cannot prosecute food street vendors who do not respect sanitary regulations. Sometimes, these activities are hidden, but more often they are done in plain sight. These activities do not fall below the state’s radar, to use Perry and collaborators’ expression (Perry et. al. 2007). Why is it that governments do not enforce labor security norms then?

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5 The effects of the government limited enforcement capacity may be compounded by individuals’ unwillingness to whistle-blow law breakers in environments of high informality (Acemoglu, 2014).
We therefore explore the idea that, in weak institutional environments, governments tolerate these activities—so they choose not to enforce some norms—, because a strict enforcement of the norms could negatively impact on individuals’ well-being. What the states are unable to do is to provide sophisticated social protection that could substitute for individuals informal risk-coping strategies. We thus emphasize the low quality of public services.

Perry et. al. (2007) and Saavedra and Tommasi (2007) also mention the low quality of public services as a driver of informality, but the specific channel is different. In their view, some individuals opt for informality—exit the social contract, in their words—, among other reasons, because of public services low quality. In our story, governments’ tolerance of non-compliance is a way out of the rigidity imposed by formal programs and policies in contexts in which the state is unable to provide more nuanced formal social policies. So we see tolerance of informality as a form of discretionary social policy.

In all the stories summarized above, the lack of state capacities is at the core of the explanation of informality, but they appeal to different capacities. Accordingly, the normative implications differ. Investing in strengthening monitoring and enforcement capacities would be advisable, if those were the issues. But if the problem lied in the inability to provide sophisticated social protection, improving monitoring and enforcement capacities might not be the solution and could even be counterproductive because they could reduce informal social protection.

The hypothesis that governments tolerate some degree of informality to protect citizens has a few antecedents in the literature. Ceni (2014), for example, argues that one possible motive for government tolerance of informality is flexibility. In the same vein, Loayza and Rigolini (2011) argue that the countercyclical behavior of informality suggests that it operates as a safety net. Our work is probably closer in aims and vision to that of Holland (2014, 2016). Analyzing the cases of street vendors and squatters in three Latin American cities, Holland provides convincing evidence that governments choose not to enforce the law as a form of protecting poor citizens. The present paper provides a model that can help think formally on these ideas.

There are of course other reasons why governments might manipulate enforcement to get political support. Sandmo (2005), analyzing the case of tax evasion, concludes that “It is,
e.g., not obvious that the low degree of enforcement of the tax law in some sectors or countries is entirely due to cost considerations; it may also be because the electorate is actually against attempts to achieve a higher rate of compliance. The reasons underlying such resistance may be several, reflecting both judgements of the overall fairness of the tax system and people’s self-interest in a lax enforcement policy, either as sellers or buyers of black market services”. Brollo et. al. (2014) show that the enforcement of conditionality in a social program in Brazil had an impact on voters behavior and that politicians manipulated the implementation of conditionality before elections. Also in the case of Brazil, Feierherd (2014) shows that big firms receive comparatively more inspections vis a vis small firms when the mayor belongs to the left-wing labor party (PT) than to the center-wing social-democratic party (PSDB). Casaburi and Troiano (2015) show that an Italian anti-tax evasion program had significant and sizable effects on electoral outcomes.

The story we tell to explain discretionary social protection policies is not the only possibility. In the literature, these policies have been associated to clientelism and patronage. Díaz-Cayeros and Magaloni (2009), referring to Latin America, argue that “For most of the twentieth century, truncated (that is, poorly targeted) welfare states coexisted with an array of clientelist transfers that did reach the poor but were subject to immense government discretion.” In this view, discretion is a natural consequence of clientelism. Dixit and Londregan (1996) provide a model of machine politics in which politicians who are unable to commit transfers and citizens who are unable to commit their vote end up trapped in a highly distortive equilibrium in which the government supports decadent industries.

Our model could also serve to think about the incomplete coverage of formal social programs in developing countries, something that is often identified as the truncated welfare state. The low coverage issue has received much attention in recent years. Multilateral organizations as the World Bank and the ILO have devoted considerable efforts to the empirical characterization of the phenomenon and have designed policy proposals to deal with this issue (see, among others, Gill et. al. 2003; Lindert et. al. 2005; Holzmann et. al. 2009; Rofman et. al. 2011; ILO 2011). If the story we tell is correct, however, part of government actions—as well as omissions—directly geared to social protection do not belong to formal public programs and are not being captured in these
analysis. The welfare states in developing countries might be less truncated, but more discretionary, than what those studies suggest.

Special interest groups and state capture are often mentioned as the main causes behind the truncated welfare state (Haggard and Kaufman 2008; Mason and Robalino 2009). Our model suggests that the welfare state might be truncated even if there were no special interest groups. This of course does not mean that special interest politics does not play a role, even a key role, in truncation. We simply argue that governments in search of flexibility might opt for truncated programs, recognizing that not all individuals fit in the formal programs they can provide.

Some of the ideas in the present paper have been advanced by Forteza (2011) in an informal essay. The main contribution of the present paper is to formalize these ideas. Appealing to a formal model, we can be much more precise regarding the meaning of what he calls “the informal welfare state”, which is our discretionary social protection regime. Formalization of the hypothesis is also crucial for a formal assessment of its internal consistency. Finally, even though we do not advance in the empirical testing, we think that formalization of the story provides better grounds to advance in that direction in the future.

In terms of the formal model, we borrow from Forteza (1999). We extend his model introducing a component of output that cannot be insured on a pre-committed basis. Because of verifiability or true uncertainty issues, the government can only commit to partially contingent rules. Under discretion, the government can insure ex-post all output. In this setting, commitment is not always better than discretion.

Three sections follow this introduction. In the second section we present the formal model. We describe the economy and solve the model under discretion and commitment. In the third section, we compute utility at the optimum and compare the pros and cons of committing to a simple social protection rule and conducting a discretionary policy. We summarize and conclude in the fourth section.

**The Model**

**The basic setting**
We consider a population of measure one. Even though the population is homogenous from an ex-ante perspective, for expository convenience, we index individuals with \( i \in [0,1] \).

Individuals produce and consume a single good. The output each individual produces is observable by everybody, including the government, but only part of it is verifiable by the judiciary and has a probability distribution function known by the government. The rest of output is non-verifiable and/or truly uncertain. Formal pre-committed government transfers -and hence government insurance- can only be conditioned on the verifiable probabilistic part of output, but ex-post discretionary insurance can be made contingent on both components. We denote by \( x_i \) and \( \varepsilon_i \) the components of output that, under commitment, are insurable and non-insurable, respectively.

Both components of output take two values, high (\( \bar{x} \) and \( \bar{\varepsilon} \)) and low (\( \underline{x} \) and \( \underline{\varepsilon} \)). Individual \( i \) gets \( \bar{x} \) with probability \( P(a_i) \) and \( \bar{\varepsilon} \) with probability \( q(a_i) \), where \( a_i \) stands for individual \( i \) effort. For simplicity, we assume that, conditional on \( a_i \), the insurable and uninsurable components of output are independent: \( \text{Prob}(x_i, \varepsilon_i | a_i) = \text{Prob}(x_i | a_i) \text{Prob}(\varepsilon_i | a_i) \).

Effort can be high (\( H \)) and low (\( L < H \)): \( a_i \in \{ H, L \} \). We define the probabilities of high output given the effort level as \( P_k = P(a_i = k) \) and \( q_k = q(a_i = k) \). The probability of high insurable output is increasing in effort \( P_H > P_L \), and of high non-insurable output is non-decreasing in effort \( q_H \geq q_L \). If \( q_H > q_L \), the unconditional distributions of \( x_i \) and \( \varepsilon_i \) are not independent, for high effort raises the probability of high output in both components.

Because of government policies, disposable income \( (w_i) \) may differ from income before transfers \( (x_i + \varepsilon_i) \). The government is constrained only by the aggregate resources constraint and a non-negativity constraint on disposable income, so we do not impose any special constraint on the form of redistributive policies. Nevertheless, the government will have no motive in our setting to treat differently two individuals who got the same output, so we will focus on policies that can be written as mappings from the pairs \( (x_i, \varepsilon_i) \) to disposable income, i.e. \( w_i = w(x_i, \varepsilon_i) \).

Individuals preferences can be represented by an expected utility function increasing and concave in consumption, decreasing in effort and, for simplicity, additively separable in
consumption and effort. In our framework, individuals will consume their disposable income, so we can directly write their expected utility in terms of income: $E[u(w(x_i, \epsilon_i))] - a_i$.

We assume a benevolent government that maximizes a social welfare function à la Bentham. Because individuals are risk averse, the government will provide insurance.

We consider two policy regimes, discretion and commitment. In a discretionary regime, individuals decide effort at the beginning. Afterwards output is realized and the government redistributes income. In this regime, the government can condition transfers on total output. Under commitment, the government must choose disposable income at the beginning. Commitment constraints the government to redistribution schemes that are contingent only on insurable output: $w_i = w(x_i)$. Afterwards, individuals choose the level of effort and output is realized.

**Discretion**

The timing in this regime is as follows:

1. Individuals play first, choosing $a_i$.
2. Nature chooses $\{x_i, \epsilon_i\}$, with probabilities $P(a_i)$ and $q(a_i)$.
3. The government chooses $w(x_i, \epsilon_i)$.

Solving by backward induction, we begin by the government problem.

In this environment, the government will condition disposable income on both components of pre-transfers income: $w_i = w(x_i, \epsilon_i)$. Hence individuals disposable income can take four values: $w(\bar{x}, \bar{\epsilon})$, $w(\bar{x}, \bar{\epsilon})$, $w(\bar{x}, \bar{\epsilon})$ and $w(x, \epsilon)$.

The population is homogenous, so in equilibrium all individuals will choose the same action: $a_i = a$. When the government turn to play arrives, a fraction $P(a)q(a)$ of citizens will have gotten $(\bar{x}, \bar{\epsilon})$, a fraction $(1 - P(a))q(a)$ will have gotten $(\bar{x}, \bar{\epsilon})$, $P(a)(1 - q(a))$ will have gotten $(\bar{x}, \epsilon)$ and a fraction $(1 - P(a))(1 - q(a))$ will have gotten $(x, \epsilon)$. We can thus write the government problem as:
\[
\max_{\{0 \leq w(x, \varepsilon)\}} \left[ q(a) \left[ P(a)u(w(x, \varepsilon)) + (1 - P(a))u\left(w(x, \varepsilon)\right) \right] + (1 - q(a)) \left[ P(a)u(w(x, \varepsilon)) + (1 - P(a))u\left(w(x, \varepsilon)\right) \right] - a \right. \\
\left. s.t. q(a) \left[ P(a)(\bar{x} + \bar{\varepsilon} - w(\bar{x}, \bar{\varepsilon})) + (1 - P(a))\left(\bar{x} + \bar{\varepsilon} - w(\bar{x}, \bar{\varepsilon})\right) \right] + (1 - q(a)) \left[ P(a)(\bar{x} + \bar{\varepsilon} - w(\bar{x}, \bar{\varepsilon})) + (1 - P(a))\left(\bar{x} + \bar{\varepsilon} - w(\bar{x}, \bar{\varepsilon})\right) \right] \right. \\
\left. \geq 0 \right]
\]

It follows from the first order conditions that the government equalizes income, providing full insurance: \( w(\bar{x}, \bar{\varepsilon}) = w(x, \varepsilon) = w(\bar{x}, \varepsilon) = w(x, \varepsilon) = w \).

At the beginning, individuals maximize utility knowing that the government will provide full insurance:

\[
\max_{a_t} u(w) - a_t
\]

Disposable income equals expected average output, which only marginally depends on individual \( i \) action, so he chooses minimum effort:

\[
a_t = L
\]

Using these results in the resources constraint:

\[
w = P_L \bar{x} + (1 - P_L)\bar{x} + q_L \bar{\varepsilon} + (1 - q_L)\varepsilon
\]

Expected individual welfare in equilibrium under discretion is thus:

\[
u(w) - L
\]

We summarize these results in the following proposition.

**Proposition 1.** Under discretion, the government provides full insurance on total output, individuals exert low effort and expected output is low: \( E[x + \varepsilon] = P_L \bar{x} + (1 - P_L)\bar{x} + q_L \bar{\varepsilon} + (1 - q_L)\varepsilon \).

**Commitment**

When the government has the ability to commit, the sequence of events is as follows:
1. The government plays first and chooses $w(x_i) \in \{\underline{w}, \overline{w}\}$, where $\underline{w} = w(x)$ and $\overline{w} = w(x)$. 

2. Individuals play later and choose $a_i$. 

3. Finally, nature plays and chooses $\{x_i, \epsilon_i\}$, with probabilities $P(a_i)$ and $q(a_i)$. 

As we will formally show below, the government option set is not convex and exhibits angles. To deal properly with these difficulties, we make extensive use of graphical solutions. A point $(\underline{w}, \overline{w})$ in the positive quadrant characterizes a redistributive policy. To be feasible, policies must respect the resources constraint which, in turn, depends on government policies.

On the 45° line, the government provides full insurance on insurable output $(\underline{w} = \overline{w})$, but individuals are not fully insured because the government is unable to condition transfers on the uninsurable component of output.

As in discretion, we solve by backward induction. We start by analyzing individuals’ choice and then we analyze the government’s problem.

**Individuals’ optimization problem**

Individuals maximize their expected utility given the insurance contract established by the government:

$$\max_{a_i} U(a_i; \underline{w}, \overline{w}) = q(a_i)[P(a_i)u(\overline{w} + \epsilon) + (1 - P(a_i))u(\underline{w} + \epsilon)]$$

$$+ (1 - q(a_i))[P(a_i)u(\overline{w} + \epsilon) + (1 - P(a_i))u(\underline{w} + \epsilon)] - a_i$$

Both effort levels report the same utility to the agent $i$ if and only if the following condition holds:

$$q_H[P_Hu(\overline{w} + \epsilon) + (1 - P_H)u(\underline{w} + \epsilon)] + (1 - q_H)[P_Hu(\overline{w} + \epsilon) + (1 - P_H)u(\underline{w} + \epsilon)]$$

$$- q_L[P_Lu(\overline{w} + \epsilon) + (1 - P_L)u(\underline{w} + \epsilon)]$$

$$- (1 - q_L)[P_Lu(\overline{w} + \epsilon) + (1 - P_L)u(\underline{w} + \epsilon)] = H - L$$

This condition represents an incentive compatibility constraint (ICC). Points that satisfy this condition belong to the locus where both effort levels report the same utility to the agent in the $(\underline{w}, \overline{w})$ space. This locus, which we name incentive compatibility line (ICL),
delimits two regions with high and low effort. We define formally these two regions as follows:

High effort region \((a_i = H)\): \(HR = \{w, \bar{w} | U(H; w, \bar{w}) \geq U(L; w, \bar{w})\}\)

Low effort region \((a_i = L)\): \(LR = \{w, \bar{w} | U(H; w, \bar{w}) < U(L; w, \bar{w})\}\)

We provide a synthetic characterization of the two regions below, leaving formal proofs to the appendix.

(A) Points to the left of the ICL belong to the high effort region and points to the right belong to the low effort region. Therefore, the ICL determines the frontier between the effort regions.

(B) Full insurance (on insurable output) is compatible with high effort if and only if non-insurable output provides enough incentives for individuals to pick high effort. If this is the case, the ICL crosses the 45° line at a strictly positive income level \(w^*\) and all full insurance policies that provide income lower than or equal to \(w^*\) induce high effort. Otherwise, the ICL lies to the left of the 45° line.

(C) The slope of the ICL can be negative, positive or infinite, but never zero. It is strictly positive if \(q_H = q_L\). We will assume in what follows that it is positive. This will be the case if increases in disposable income in the good state of nature (\(\bar{w}\)) induce increases in effort, which looks natural (see remark 2 in the appendix).

Figure 1 provides representations of the effort regions. In the left panel, we consider a case in which full insurance induces low effort. In the right panel, we represent an example in which the ICL crosses the 45° line and full insurance is consistent with high effort, provided after-transfer income is not too large.
**Government’s problem**

The benevolent government maximizes an expected welfare function à la Bentham, subject to the economy resources constraint and to the individuals incentive compatibility constraints.

\[
\max_{\overline{w},\underline{w}} \int \left[ q(a_i) \left( P(a_i)u(\overline{w} + \bar{\varepsilon}) + (1 - P(a_i))u(\underline{w} + \bar{\varepsilon}) \right) 
+ \left( 1 - q(a_i) \right) \left( P(a_i)u(\underline{w} + \varepsilon) + (1 - P(a_i))u(\overline{w} + \varepsilon) \right) - a_i \right] di \\
\text{s.t. } \int \left[ P(a_i)(\overline{\chi} - \overline{w}) + (1 - P(a_i))(\underline{\chi} - \underline{w}) \right] di \geq 0 \\
a_i = \arg\max_a U(a; w, \overline{w}) \quad \forall i
\]

We solve the program in stages. First, we find the optimal policy for given effort levels. Then, according to the previous results, we compare utility levels reached with each policy in both effort levels to determine the government’s best policy.

The slope of the indifference curves (U) and the resources constraint (RC) are:
\[
\frac{d\bar{w}}{d\bar{w}}_{\bar{w}} = -\frac{(1 - P(a_i))}{P(a_i)} q(a_i)u'(\bar{w} + \bar{\varepsilon}) + (1 - q(a_i))u'(\bar{w} + \bar{\varepsilon}) < 0
\]

\[
\frac{d\bar{w}}{d\bar{w}}_{RC} = -\frac{(1 - P(a_i))}{P(a_i)} < 0
\]

The signs can be easily verified using that: (i) \(0 < u'(.)\); (ii) \(0 \leq P(a_i) \leq 1\); and (iii) \(0 \leq q(a_i) \leq 1\).

The indifference curves are flatter than the resources constraint to the right of the 45° line, tangent to the resources constraint on the 45° line, and steeper than the resources constraint to the left of the 45° line: \(^6\)

\[
\frac{d\bar{w}}{d\bar{w}}_{\bar{w}} > \frac{d\bar{w}}{d\bar{w}}_{RC} \text{ if } \bar{w} > \bar{w}
\]

\[
\frac{d\bar{w}}{d\bar{w}}_{\bar{w}} < \frac{d\bar{w}}{d\bar{w}}_{RC} \text{ if } \bar{w} = \bar{w}
\]

The slopes of the indifference curves and the resources constraint are discontinuous at the points where these curves cross the ICL. In a small enough neighborhood of the crossing, the resources constraint and the indifference curves are strictly steeper in the low than in the high effort region. \(^7\) The resources constraint also exhibits a discontinuity in level at the crossing, unless the no-redistribution point \((\bar{x}, \bar{x})\) lies on the ICL.

The government maximizes social welfare choosing the indifference curve that is farthest from the origin. Conditional on high effort, the best the government can do is to pick the pair \((\bar{w}, \bar{w})\) at the crossing of the resources constraint with the ICL, if the crossing takes place to the left of the 45° line, and at the crossing of the resources constraint and the 45° line, otherwise. Conditional on low effort, the government optimizes picking the pair \((\bar{w}, \bar{w})\) at the crossing of the resources constraint with the ICL, if it takes place to the right of the 45° line, and at the crossing of the resources constraint and the 45° line, otherwise. These results follow immediately from the properties of the indifference curves and the resources constraint described above. Figure 2 summarizes the government possible optimal responses given the effort level.

\(^6\) These results can be verified using that: (i) \(0 < u'(.)\); (ii) \(u'(\bar{w} + \bar{\varepsilon}) > u'(\bar{w} + \bar{\varepsilon})\) if \(\bar{w} < \bar{w}\), and \(u'(\bar{w} + \bar{\varepsilon}) < u'(\bar{w} + \bar{\varepsilon})\) if \(\bar{w} > \bar{w}\); (iii) \(u'(\bar{w} + \bar{\varepsilon}) > u'(\bar{w} + \bar{\varepsilon})\) if \(\bar{w} < \bar{w}\), and \(u'(\bar{w} + \bar{\varepsilon}) < u'(\bar{w} + \bar{\varepsilon})\) if \(\bar{w} > \bar{w}\); and (iv) \(0 \leq q(a_i) \leq 0 \rightarrow 0 \leq (1 - q(a_i)) \leq 1\).

\(^7\) These results can be verified using that: (i) \(P_L < P_H \rightarrow (1 - P_L) > (1 - P_H)\); and (ii) \(0 < u'(.)\).
Using the above observations, we can characterize the set of equilibria in a commitment regime as follows:

**Proposition 2**: Under commitment, three types of equilibria may arise, depending on parameter values: (i) full insurance (on insurable output) and low effort; (ii) incomplete insurance and high effort; and (iii) full insurance (on insurable output) and high effort. There is no equilibrium with incomplete insurance (on insurable output) and low effort.

**Proof**: In figure 3, we show examples of the three mentioned types of equilibria to prove that they may exist. Which equilibrium arises depends on parameter values.
We now prove that there is no equilibrium with incomplete insurance (on insurable output) and low effort. We have already shown (figure 2) that the optimal government response in the low effort region involves full insurance (on insurable output) if this is feasible, i.e. if the resources constraint crosses the 45° line in LR. We have also shown that, in the opposite case, the best the government can do in the low effort region is to pick the point at
the crossing of the low-effort branch of the resources constraint and the ICL, a point in which the government does not provide full insurance (point A in the last panel in figure 3). This seems to contradict our hypothesis, but we now show that this point will not be chosen since other feasible points deliver higher expected welfare. In particular, the point at the crossing of the high-effort branch of the resources constraint and the ICL is feasible and yields higher expected welfare (point B in the last panel in figure 3).

Individuals expected utility at points A and B are:

\[
U(L; w_A, \bar{w}_A) = q_L [P_L u(\bar{w}_A + \varepsilon) + (1 - P_L) u(w_A + \varepsilon)] \\
+ (1 - q_L) [P_L u(\bar{w}_A + \varepsilon) + (1 - P_L) u(w_A + \varepsilon)] - L
\]

\[
U(H; w_B, \bar{w}_B) = q_H [P_H u(\bar{w}_B + \varepsilon) + (1 - P_H) u(w_B + \varepsilon)] \\
+ (1 - q_H) [P_H u(\bar{w}_B + \varepsilon) + (1 - P_H) u(w_B + \varepsilon)] - H
\]

On the ICL, individuals are indifferent between low and high effort:

\[
U(H; w_B, \bar{w}_B) = U(L; w_A, \bar{w}_A)
\]

And we can write that:

\[
U(H; w_B, \bar{w}_B) - U(L; w_A, \bar{w}_A)
= q_L \left[ P_L (u(\bar{w}_B + \varepsilon) - u(\bar{w}_A + \varepsilon)) + (1 - P_L) \left( u(\bar{w}_B + \varepsilon) - u(w_A + \varepsilon) \right) \right] \\
+ (1 - q_L) \left[ P_L (u(\bar{w}_B + \varepsilon) - u(\bar{w}_A + \varepsilon)) + (1 - P_L) (u(w_B + \varepsilon) - u(w_A + \varepsilon)) \right] > 0
\]

Where the inequality holds because (i) \(w_B > w_A\), and (ii) \(\bar{w}_B > \bar{w}_A\), since the ICL has positive slope. QED.

**The dilemma of formalization**

A government that can abandon discretion to embrace formal pre-committed social protection policies may face a non-trivial choice if fully contingent rules are not possible. To clarify the relevant tradeoffs, we compare utility in equilibrium under discretion and commitment.
Discretion versus commitment with full insurance and low effort

Under discretion, expected utility is:

\[ u(x_L + \epsilon) - L \]

Where \( x_L = P_L \bar{x} + (1 - P_L)x \) and \( \epsilon_L = q_L \bar{\epsilon} + (1 - q_L)\epsilon \).

In a commitment equilibrium with full insurance and low effort, expected utility is:

\[ q_L u(x_L + \bar{\epsilon}) + (1 - q_L)u(x_L + \epsilon) - L \]

We compute the welfare gain of formalization as:

\[ G = q_L u(x_L + \bar{\epsilon}) + (1 - q_L)u(x_L + \epsilon) - u(x_L + \epsilon_L) < 0 \]

Where the inequality above follows from Jensen’s inequality.

Therefore, discretion is better than commitment if the commitment equilibrium involves low effort. In this case, formalization of social policies (committing) brings no efficiency gain and entails some loss in terms of reduced insurance due to the inability of the government to insure all output with formal programs.

Notice the key role that the inability of the government to commit to a fully contingent policy plays in this result: if all output were insurable the government could always commit to a policy that mimics the policy that is optimal under discretion. In this case, \( \epsilon = \bar{\epsilon} = \epsilon_L = 0 \) and \( G = 0 \). Discretion would never be better than commitment. But the reverse would not be true: there would be some policies that could be implemented under commitment but not under discretion. Therefore, if output were fully insurable under commitment, discretion would be (weakly) dominated by commitment.

Discretion versus commitment with incomplete insurance and high effort

Expected utility under commitment is:

\[
U(H; \underline{w}^*, \bar{w}^*) = q_H [P_H u(\bar{w}^* + \bar{\epsilon}) + (1 - P_H)u(\underline{w}^* + \bar{\epsilon})] \\
+ (1 - q_H) [P_H u(\underline{w}^* + \epsilon) + (1 - P_H)u(\underline{w}^* + \epsilon)] - H
\]
Where the pair \((w^*, \bar{w}^*)\) is determined by the crossing of the ICL and the high-effort branch of the resources constraint (panel B in figure 3).

The welfare gain from formalization is:

\[ G = U(H; w^*, \bar{w}^*) - u(x_L + \varepsilon_L) + L \]

Doing some manipulation that we leave to the appendix, we approximate the welfare gain from formalization as follows:

\[ G \cong u'(x_L + \varepsilon_L)(x_H + \varepsilon_H - x_L - \varepsilon_L) + \frac{1}{2} u''(x_L + \varepsilon_L)E[(\Delta w + \Delta \varepsilon)^2] - (H - L) \]

Where the expected square gains (and losses) from commitment are:

\[ E[(\Delta w + \Delta \varepsilon)^2] = q_H P_H \left( \bar{w}^* + \bar{\varepsilon} - x_L - \varepsilon_L \right)^2 + q_H (1 - P_H) \left( \bar{w}^* + \bar{\varepsilon} - x_L - \varepsilon_L \right)^2 + \]

\[(1 - q_H) P_H \left( \bar{w}^* + \bar{\varepsilon} - x_L - \varepsilon_L \right)^2 + (1 - q_H)(1 - P_H) \left( w^* + \varepsilon - x_L - \varepsilon_L \right)^2 > 0 \]

The utility gain is thus decomposed in three terms. The first one is positive and captures the gain due to increased expected output. The second term is negative and captures the utility loss due to increased risk. The third term is negative and captures the utility loss due to increased effort. Committing is welfare improving only if the output gain outweighs the welfare impact of increased risk and higher effort.

The efficiency gain of committing to a formal welfare state stems in our model from the fact that, under discretion, the benevolent government faces a Samaritan’s dilemma (Buchanan, 1975). Under discretion, individuals choose to exert low effort, knowing that the government will later equalize disposable income. By committing not to provide full insurance, the government can provide better incentives. The counterpart is that individuals face higher risk.

**Discretion versus commitment with full insurance and high effort**

In a commitment equilibrium with high effort and full insurance, expected utility is:
\[ q_H u(P_H \overline{x} + (1 - P_H)\overline{x} + \overline{\epsilon}) + (1 - q_H)u(P_H \overline{x} + (1 - P_H)\overline{x} + \overline{\epsilon}) - H \]

The welfare gain of formalization is:

\[ G = q_H u(x_H + \overline{\epsilon}) + (1 - q_H)u(x_H + \epsilon) - u(x_L + \epsilon_L) - (H - L) \]

Where \( x_L \) and \( \epsilon_L \) are defined as before and \( x_H = P_H \overline{x} + (1 - P_H)\overline{x} \) and \( \epsilon_H = q_H \overline{\epsilon} + (1 - q_H)\epsilon \).

Proceeding in a similar way as in the previous subsection, we get the following approximation of the welfare gain to be expected from formalization to a commitment regime with full insurance and high effort (see the appendix for the details):

\[ G \cong u'(x_L + \epsilon_L)(x_H + \epsilon_H - x_L - \epsilon_L) \]
\[ + \frac{1}{2} u''(x_L + \epsilon_L) \left( q_H(x_H + \overline{\epsilon} - x_L - \epsilon_L)^2 \right) \]
\[ + (1 - q_H)(x_H + \epsilon - x_L - \epsilon_L)^2 \] - (H - L)

The utility gain from formalization is again decomposed in three terms with the same interpretation as in the previous case.

We summarize the results in this section in the following proposition.

**Proposition 3.** If output is not fully insurable under commitment, discretion is better than commitment if the commitment equilibrium involves low effort. Even if committing social policies brings about an efficiency gain in terms of higher output, discretion may still be preferred if the utility gains from higher expected output do not outweigh the utility losses due to higher risk and effort.

**Summary and conclusions**

We present a simple model aimed at explaining why governments often seem to provide social protection on a discretionary basis. Unable to commit to sophisticated fully contingent rules, governments may find it optimal to avoid commitment to simple rules and choose discretion in order to gain flexibility. The discretionary social protection regime described in this paper could be thought of as a stylized representation of a wide range of real world policies, including tolerance of informality, soft-budgeting and
protection of decadent industries. In this perspective, several policies that have been traditionally thought of as the result of government failures could be given a different rationale.

We elaborate on a model of social insurance presented by Forteza (1999). Incorporating a non-insurable component of output, we provide a simple representation of the idea that governments may not be able to condition formal social protection policies on total individual output. Because of this, governments cannot commit to fully contingent policies and are constrained in the amount of insurance they can provide on a pre-committed basis. We call these pre-committed partially contingent policies simple rules.

Governments can decide not to tie their hands, choosing transfers after observing outcomes. In so doing, they gain flexibility in the sense that they can condition ex-post transfers on all observable outcomes, disregarding issues related to non-verifiability and/or Knightian uncertainty. This discretionary social protection regime is our formal representation of what Forteza (2011) named “the informal welfare state”.

In equilibrium, the discretionary regime is characterized by low effort and full insurance. The commitment regime, in turn, exhibits three possible equilibria. One in which the government provides full insurance on insurable output and individuals choose low effort. There is a second equilibrium in which the government provides incomplete insurance and individuals choose high effort. Finally, there is an equilibrium characterized by full insurance on insurable output and high effort. In this last equilibrium, the non-insurable component of output provides the incentives for individuals to choose high effort.

Comparing expected utility in equilibrium, we show that commitment to a simple rule is better than discretion if the efficiency gain due to lesser distortions outweighs the loss of flexibility. We identify three potential effects on welfare of shifting from discretion to commitment to a simple rule: a positive effect due to increased output, a negative effect due to higher risk and a negative effect associated to higher effort. Only when the first effect dominates will commitment improve on discretion. This does not happen, for example, if the commitment equilibrium involves low effort. Moving from discretion to commitment in this case would entail losing insurance without gaining efficiency.

We conclude that governments may rationally choose discretion rather than rules if they are unable to commit to fully contingent social protection policies. In weak institutional
environments, tolerance of informality, repeated bail-outs, soft-budgeting, protection of decadent industries and a large etcetera that represent different forms of discretion could be rationalized on these grounds. The real challenge in these cases is to strengthen state capacities in ways that make it possible to commit to sufficiently sophisticated social policies so that the informal welfare state can be phased out, giving place to more formal policies.
References


Appendix

1. Characterization of the ICL and the effort regions

(A) Points to the left (right) of the ICL are in the high (low) effort region.

Proof:

\[
\frac{\partial U}{\partial w}(a; \overline{w}, w) = q_a(1 - P_a)u'(w + \varepsilon) + (1 - q_a)(1 - P_a)u'(w + \varepsilon)
\]

\[
\Rightarrow \frac{\partial U}{\partial w}(H; \overline{w}, w) - \frac{\partial U}{\partial w}(L; \overline{w}, w) = [q_H(1 - P_H) - q_L(1 - P_L)]u'(w + \varepsilon) + [(1 - q_H)(1 - P_H) - (1 - q_L)(1 - P_L)]u'(w + \varepsilon) = (1 - P_H)[q_Hu'(w + \varepsilon) + (1 - q_H)u'(w + \varepsilon)] < 0
\]

The last inequality is based on the following observations:

(i) \( u'(w + \varepsilon) < u'(w + \varepsilon) \)

(ii) \( q_H > q_L \)

\[
[q_Hu'(w + \varepsilon) + (1 - q_H)u'(w + \varepsilon)] < [q_Lu'(w + \varepsilon) + (1 - q_L)u'(w + \varepsilon)]
\]

(iii) \( P_H > P_L \Rightarrow (1 - P_H) < (1 - P_L) \)

(B) The ICL crosses the 45° line iff the non-insurable component of output induces high effort. Let \( w^* \) be the after-transfers income at the crossing. Individuals exert high effort if the government provides full insurance with income levels not larger than \( w^* \).

Proof: The ICL intersects the 45° line at \( w = \overline{w} = w^* \) iff there is a \( w^* \) such that:

\[
(q_H - q_L)[u(w^* + \varepsilon) - u(w^* + \varepsilon)] = H - L
\]

We define the function \( f(w; \overline{w}, \varepsilon) \):

\[
f(w; \overline{w}, \varepsilon) = u(w + \varepsilon) - u(w + \varepsilon)
\]

It is immediately obvious that \( f(w; \overline{w}, \varepsilon) > 0 \) and \( f'(w; \overline{w}, \varepsilon) < 0 \).

Therefore, iff \( \left( 0; \overline{w}, \varepsilon \right) > \frac{H - L}{q_H - q_L} \), there is one and only one \( w^* > 0 \), defined by equation (3), such that the ICL and the 45° line intersect. This condition can be written as follows:
\( q_H u(\tilde{e}) + (1 - q_H)u(\varepsilon) - H > q_L u(\tilde{e}) + (1 - q_L)u(\varepsilon) - H \). If this inequality holds, we say that non-insurable output induces high effort in the sense that even when the government provides maximum insurance on the insurable component individuals exert high effort. It is immediately obvious that individuals choose high effort for any policy such that \( w = \underline{w} = \bar{w} \leq w^* \).

(C) The slope of the ICL.

We totally differentiate the ICL to analyze its slope:

\[
\left[ (q_H P_H - q_L P_L) u'(\bar{w} + \tilde{e}) + ((1 - q_H)P_H - (1 - q_L)P_L)u'(\bar{w} + \varepsilon) \right] d\bar{w} \\
+ \left[ (q_H (1 - P_H) - q_L (1 - P_L)) u'(\bar{w} + \tilde{e}) + ((1 - q_H)(1 - P_H) - (1 - q_L)(1 - P_L))u'(\bar{w} + \varepsilon) \right] d\bar{w} = 0
\]

\[
\frac{d\bar{w}}{dw}_{ICL} = \frac{A u'(\bar{w} + \tilde{e}) + Bu'(\bar{w} + \varepsilon)}{C u'(\bar{w} + \tilde{e}) + Du'(\bar{w} + \varepsilon)}
\]

Where:

\[
A = (q_L (1 - P_L) - q_H (1 - P_H)) \\
B = (1 - q_L)(1 - P_L) - (1 - q_H)(1 - P_H) \\
C = q_H P_H - q_L P_L \\
D = (1 - q_H)P_H - (1 - q_L)P_L
\]

We now show some attributes of the ICL that can be derived from this expression.

**Remark 1:** the numerator of the slope of the ICL is positive.

**Proof:**

\[
A u'(\bar{w} + \tilde{e}) + Bu'(\bar{w} + \varepsilon) \\
= (q_L (1 - P_L) - q_H (1 - P_H))u'(\bar{w} + \tilde{e}) \\
+ ((1 - q_L)(1 - P_L) - (1 - q_H)(1 - P_H))u'(\bar{w} + \varepsilon) \\
= (1 - P_L) \left( q_L u'(\bar{w} + \tilde{e}) + (1 - q_L)u'(\bar{w} + \varepsilon) \right) \\
- (1 - P_H) \left( q_H u'(\bar{w} + \tilde{e}) + (1 - q_H)u'(\bar{w} + \varepsilon) \right) > 0
\]

The above inequality stems from the following observations:

(i) \( u'(\bar{w} + \tilde{e}) < u'(\bar{w} + \varepsilon) \)

(ii) \( q_H > q_L \)

\[
\Rightarrow [q_L u'(\bar{w} + \tilde{e}) + (1 - q_L)u'(\bar{w} + \varepsilon)] > [q_H u'(\bar{w} + \tilde{e}) + (1 - q_H)u'(\bar{w} + \varepsilon)]
\]
Remark 2: The denominator of the slope of the ICL can be positive, zero or negative. This implies, together with remark 1, that the slope of the ICL can be positive, infinite or negative. The denominator and the slope of the ICL are strictly positive if \( q_H = q_L \).

The denominator can be written as:

\[
Z = P_H \left[ q_H u'(\bar{w} + \bar{\varepsilon}) + (1 - q_H)u'(\bar{w} + \bar{\varepsilon}) \right] - P_L \left[ q_L u'(\bar{w} + \bar{\varepsilon}) + (1 - q_L)u'(\bar{w} + \bar{\varepsilon}) \right]
\]

The terms within brackets are the expected marginal utilities of \( \bar{w} \) when individuals choose high and low effort. If \( q_H > q_L \), the marginal utility is higher with low than high effort:

\[
q_H u'(\bar{w} + \bar{\varepsilon}) + (1 - q_H)u'(\bar{w} + \bar{\varepsilon}) < q_L u'(\bar{w} + \bar{\varepsilon}) + (1 - q_L)u'(\bar{w} + \bar{\varepsilon})
\]

which makes the sign of \( Z \) ambiguous. If instead \( q_H = q_L \), the marginal utility of \( \bar{w} \) does not depend on effort and \( Z > 0 \).

An increase in disposable income in the good state of nature (\( \bar{w} \)) provides incentives to choose high effort, since the probability of the good state of nature is higher with high than low effort (\( P_H > P_L \)). However, there is also an indirect effect that acts in the opposite direction if effort raises the probability of both insurable and non-insurable output (\( q_H > q_L \)). In this case, an increase in effort reduces the marginal utility of \( \bar{w} \) because it increases the probability of high realizations of the uninsurable component of output. An increase in \( \bar{w} \) has a positive effect on effort, and the slope of the ICL is positive, if and only if the direct effect outweighs the indirect effect. We assume this is the case for all the relevant values of \( \bar{w} \).

2. Derivation of equation (1)

The welfare gain from formalization when the commitment equilibrium entails incomplete insurance and high effort is:

\[
G = q_H \left[ P_H u(\bar{w}^* + \bar{\varepsilon}) + (1 - P_H)u(\bar{w}^* + \bar{\varepsilon}) \right] + (1 - q_H) \left[ P_H u(\bar{w}^* + \bar{\varepsilon}) + (1 - P_H)u(\bar{w}^* + \bar{\varepsilon}) \right] - H - u(x_L + \varepsilon_L) + L
\]

Taylor series expansion:

\[
u(\bar{w}^* + \bar{\varepsilon}) = u(x_L + \varepsilon_L) + u'(x_L + \varepsilon_L)(\bar{w}^* + \bar{\varepsilon} - x_L - \varepsilon_L)
+ \frac{1}{2} u''(x_L + \varepsilon_L)(\bar{w}^* + \bar{\varepsilon} - x_L - \varepsilon_L)^2
\]

Doing analogous expansions for the other three terms, substituting and rearranging:
$$G \equiv q_H P_H \left[ u'(x_L + \varepsilon_L)(\bar{w}^* + \bar{\varepsilon} - x_L - \varepsilon_L) + \frac{1}{2} u''(x_L + \varepsilon_L)(\bar{w}^* + \bar{\varepsilon} - x_L - \varepsilon_L)^2 \right]$$

$$+ q_H (1 - P_H) \left[ u'(x_L + \varepsilon_L)(\bar{w}^* + \bar{\varepsilon} - x_L - \varepsilon_L) + \frac{1}{2} u''(x_L + \varepsilon_L)(\bar{w}^* + \bar{\varepsilon} - x_L - \varepsilon_L)^2 \right]$$

$$+ (1 - q_H) P_H \left[ u'(x_L + \varepsilon_L)(\bar{w}^* + \bar{\varepsilon} - x_L - \varepsilon_L) + \frac{1}{2} u''(x_L + \varepsilon_L)(\bar{w}^* + \bar{\varepsilon} - x_L - \varepsilon_L)^2 \right]$$

$$+ (1 - q_H)(1 - P_H) \left[ u'(x_L + \varepsilon_L)(\bar{w}^* + \bar{\varepsilon} - x_L - \varepsilon_L) + \frac{1}{2} u''(x_L + \varepsilon_L)(\bar{w}^* + \bar{\varepsilon} - x_L - \varepsilon_L)^2 \right] - (H - L)$$

The policy $(\bar{w}^*, \bar{w}^*)$ is on the high effort branch of the resources constraint and hence:

$$P_H \bar{w}^* + (1 - P_H)w^* = P_H \bar{x} + (1 - P_H)x = x_H$$

Substituting this expression in $G$ and operating, we get equation (1).

3. Derivation of equation (2)

The welfare gain from formalization when the commitment equilibrium entails full insurance and high effort can be written as:

$$G = q_H \left( u(x_H + \bar{\varepsilon}) - u(x_L + \varepsilon_L) \right) + (1 - q_H) \left( u(x_H + \varepsilon_L) - u(x_L + \varepsilon_L) \right) - (H - L)$$

Using a Taylor series expansion:

$$u(x_H + \bar{\varepsilon}) \equiv u(x_L + \varepsilon_L) + u'(x_L + \varepsilon_L)(x_H + \bar{\varepsilon} - x_L - \varepsilon_L) + \frac{1}{2} u''(x_L + \varepsilon_L)(x_H + \bar{\varepsilon} - x_L - \varepsilon_L)^2$$

$$u(x_H + \varepsilon_L) \equiv u(x_L + \varepsilon_L) + u'(x_L + \varepsilon_L)(x_H + \varepsilon - x_L - \varepsilon_L) + \frac{1}{2} u''(x_L + \varepsilon_L)(x_H + \varepsilon - x_L - \varepsilon_L)^2$$

Substituting back in the formalization gain function, we get equation (2).