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in a Dual Currency Economy

Eduardo Siandra

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Eduardo Siandra
Departamento de Economía (FCS)
Universidad de la República
Montevideo, Uruguay
E-Mail: eduardos@decon.edu.uy
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Abstract

This paper discusses moral hazard problems in credit markets where two convertible currencies circulate. Mismatching is viewed as a moral hazard phenomenon where risk neutral borrowers gamble on the depreciation of the loan currency when interest rates are high enough. We study instances in which exchange risk raises the overall credit risk of investment projects. JEL Classification: G1 & E4.

Sumario (español): Este trabajo discute problemas de riesgo moral en mercados crediticios en donde circulan libremente varias monedas convertibles. Los descalces son vistos como un fenómeno de riesgo moral en donde tomadores de fondos neutrales al riesgo apuestan a la depreciación de la moneda del préstamo cuando las tasas de interés son lo suficientemente altas. Aquí ilustramos situaciones en las cuales el riesgo cambiario incrementa el riesgo crediticio de los proyectos de inversión.

Introduction

The central role of imperfect information, whether be about agent’s types or agent’s actions, in the workings of credit markets is widely acknowledged in the mainstream literature. For instance, Freixas-Rochet [1996]’s textbook
highlights a great deal asymmetric information problems in credit banking. Myers [1977]'s seminal paper on "underinvestment problem" hinges on conflicts of interest between bondholders and shareholders, where moral hazard is central. Lastly, Stiglitz-Weiss [1981] initiated a strand of literature on the possibility of credit rationing due to adverse selection in a population with heterogeneous risky borrowers. This intriguing non market clearing phenomenon has been extensively explored and a good, albeit old, survey is Jaffe-Stiglitz [1990].

All this research has focused in economies in which there is only one currency or money does not matter. Although there is a long tradition of analysis of credit markets in the context of a barter economy, in everyday life money and credit have been closely intertwined.

The key role of money is particularly stark in economies in which circulate several fully convertible currencies. Examples of this are Asian and Latin American countries which lifted exchange controls as a part of macrostabilization policies. In these cases a relatively weak domestic currency circulate along with a strong foreign currency. It is normally the case that the simple uncovered interest rate parity does not explained the wedge between the cost of foreign currency denominated loans and the cost of domestic currency denominated loans. It is common that domestic interest rates are much higher than what naive uncovered interest parities would suggest. Also, the relative costs of foreign and domestic denominated loans depends in intriguing fashion on the borrowers rating.

Much of the existing literature on spreads between interest rates of domestic and foreign loans and on "dollarization" of loans and deposits is of macroeconomic nature. An interesting recent work, Catao and Terrones [2000], studies this type of phenomena using a partial equilibrium model of the banking industry. They highlight the role of the credit market structure where banks play an essential role in a dual currency subject to macro shocks.

The purpose of this paper is to get further insights from a microeconomic approach, and, in particular, to explore the effects of moral hazard in the selection of the currency funding and eventual interactions with investment project decisions in an environment where the denomination of the loans is endogenous. Unlike the richness of institutional detail, we resort to a more abstract setup with no financial intermediation.

Currency mismatching is generated by risk neutral borrowers who take on a gamble on the loan currency depreciation. In this case exchange risk makes a difference as to the credit risk of the investment projects. If already there was a moral hazard problem independently of the funding currency choice, the latter one adds to the former. Risk averse lenders do not change
this outcome. The role of optimal contracts and the nature of the market incompleteness underlying this problem is left for future work.

The outline of the paper is as follows: first we present a simple moral hazard story of selection of exclusive ”good” and ”bad” projects from Bester-Hellwig [1987] in a one currency model. Next we set up a very simple two currency economy where firms have non exclusive projects in foreign and domestic currency, and the issue is the denomination of the loan. Lastly, we integrate both problems: exclusive projects and the selection of the funding currency, and convey instances in which both types of moral hazards are compounded. All diagrams are gathered at the end of the paper.

1 A simple model of credit and moral hazard

Our point of departure is a slight modification of Bester-Hellwig [1987], as summarized in Freixas-Rochet [1996]. Their model captures the limited involvement of lenders in the management of borrowing firms as a leading reason of moral hazard in investment project selection.

The story is very simple. Each firm has a choice between a ”good” project, which generates a cashflow $y_g$ with a probability $p_g$ and nothing otherwise, and a ”bad” project generating a cashflow of $y_b$ with probability $p_b$ and zero otherwise. Cashflows are defined per unit of investment.

In this setup the good project has higher expected return, $p_g y_g > p_b y_b$, and is safer, $p_g > p_b$ and $y_b > y_g$. That is, the bad project has a smaller probability of a higher cashflow, but it does not suffice for overtaking the good project expected return.

With a size loan of one, the amount to be repaid is $R$, i.e., one plus the interest rate. However, albeit sloppy, $R$ will be referred as the interest rate. Risk neutral borrowers choose the good project as long as its expected return net of repayment is higher

$$p_g(y_g - R) \geq p_b(y_b - R).$$

We can define a ”cutoff” repayment $\hat{R}$

$$\hat{R} = \frac{(p_g y_g - p_b y_b)}{(p_g - p_b)},$$

where $R \in (0, y_b)$. Thus, the good project gets chosen if and only if the market interest rate $R$ does not exceed $\hat{R}$. Notice that the moral hazard problem becomes more severe the larger the difference in cashflows $y_b - y_g$. 


and the smaller the difference in probabilities \( p_g - p_b \). The value of the firm for the owners is given for the following weakly convex function:

\[
V(R) = \max\{p_g(y_g - R), p_b(y_b - R), 0\},
\]

which is illustrated in Fig. 1 of the Appendix.

The easiest way of closing the model is to postulate a large number of perfectly competitive lenders with a given outside option. If they are risk neutral, the option can be an investment whose expected return is \( \varrho \), at which there is perfectly elastic supply of funds. Provided that \( \varrho \leq \max\{p_g\hat{R}, p_b y_b\} \), there will be at least one equilibrium interest rate \( \hat{R} \). Fig. 2 depicts the credit market equilibria. The is as follows

- There is a unique equilibrium with borrowers selecting the good project such as point A in Fig. 2 if the value of the outside option is low enough (\( \varrho < p_b\hat{R} \)).
- There may be a unique equilibrium where Borrowers select the bad project if the outside option is high enough \( \varrho > p_g\hat{R} \) and \( p_b y_b \geq p_g\hat{R} \) (point C in Fig. 2).
- One type of multiple equilibria as illustrated in points B1 and B2 in Fig. 2 for intermediate values of outside option (\( p_b\hat{R} < \varrho < p_b y_b \)).

Bester-Hellwig [1987] highlight the possibility of credit rationing when lenders are price setters and the maximum expected lending rate is achieved at the "cutoff" rate of \( \hat{R} \) (\( p_g\hat{R} > p_b y_b \)). This would give rise to a backward bending funds supply with an interior maximum at \( \hat{R} \), which, in turn, if lower than demand, then some type of rationing would emerge. But in this paper we will not pursue this line of argument any further.

The equilibrium interest rates reflect the default probabilities in a risk neutral world. Nothing would change very much if the population of lenders would display a homogeneous attitudes toward risk such as risk aversion or risk taking. But in the following section the results actually rests on different assumptions in that regard.

2 Currency mismatching as moral hazard

Here we will consider a simple credit market in an economy in which two fully convertible currencies circulate. One currency will be labeled as "domestic"
(d) and the other as "foreign" (f). The rate of exchange will be quoted as the domestic price of a unit of foreign currency. At the outset both currencies exchange at a rate of one to one. At the end the domestic price of one unit of foreign currency is a strictly positive random variable $\tilde{e}$. With an exogenously given probability $\pi$ the domestic currency will depreciate $\tilde{e} = e > 1$, and with probability $1 - \pi$ the currency will appreciate $\tilde{e} = 1/e < 1$. Changing the point of view, the foreign currency will depreciate $\tilde{e} = e > 1$ with probability $1 - \pi$ (rather than $\pi$) and appreciate $\tilde{e} = 1/e < 1$ with probability $\pi$. The effect of an increase in the parameter $e$, a measure of volatility, on the expected future exchange rate $E(\tilde{e})$ depends obviously on $\pi$. We should expect $\frac{\partial E(\tilde{e})}{\partial e} > 0$ if $\pi > 1/2$.

The choice of values of the future exchange rates is a bit contrived, but it fits nicely and simply in our setup. The equilibrium rates of return on foreign and domestic assets will convalidate the exchange rates assumed for the initial and final period.

There are two types of projects in the economy. The "foreign" project (f) generates a cashflow denominated in foreign currency $y_f$ with probability $p_f$ and nothing otherwise. In the same fashion the "domestic" project (d) produces a cashflow denominated in domestic currency $y_d$ with probability $p_d$ and zero otherwise. Probabilities $p_f$ and $p_d$ are independent of the distribution of the exchange rate $\tilde{e}$. It will simplify quite a lot to assume something like $e = y_d/y_f$. Each project requires one unit of investment in the corresponding currency, and must be fully funded with debt in order to be undertaken.

Let us define $R^i_j$ as the interest rate in currency $i$ channeled to sector $j$. So here creditors are able to determine the project and the currency. Under risk neutrality, the investment is made if the expected return of the project net debt repayment is non negative, that is, they are not exclusive.

There is currency matching on the borrowing side when projects are funded with a loan denominated in the same currency as their cashflows. In this case projects are undertaken if

$$p_f(y_f - R^f_f) \geq 0 \text{ and } p_d(y_d - R^d_d) \geq 0,$$

or $y_f \geq R^f_f$ and $y_d \geq R^d_d$.

Alternatively, borrowers could take non matching loans. In such a case the conditions for project acceptance would be more complex. Suppose we are considering the domestic project. For relatively low values of the interest rates $R^d_d$ the expected net values are non negatives regardless of the future exchange rates:

$$p_d(y_d - R^d_d e) \geq 0 \text{ and } p_d(y_d - R^d_d/e) \geq 0,$$
or equivalently, $R^d_f \leq y_d \min\{e, 1/e\} = y_d/e$. Taking expected values across future exchange rates, we obtain

$$p_d(y_d - R^d_f E(\tilde{e})) \geq 0.$$  

If the interest rates is in the upper ladder, $y_d/e < R^d_f \leq y_de$, then with probability $1 - \pi$ the domestic (foreign) currency appreciates (depreciates) and the project payoff will be

$$p_d(y_d - R^d_f/e) \geq 0.$$

Let $V_d(R^d_f)$ be the best expected net value of the domestic project as a function of the interest rate of a non matching loan:

$$V_d(R^d_f) = \begin{cases} 
\begin{align*}
p_d(y_d - R^d_f E(\tilde{e})) &\geq 0 & \text{if } R^d_f \leq y_d/e, \\
(1 - \pi)p_d(y_d - R^d_f/e) &\geq 0 & \text{if } y_d/e < R^d_f \leq y_de.
\end{align*}
\end{cases}$$

The function $V_d(R^d_f)$ is piecewise linear, continuous, weakly convex, and monotonically decreasing with a kink at $R^d_f = y_d/e$, very much like the one in Fig. 1. The expected repayment value function of the loans in foreign currency for the domestic projects are is

$$L_d(R^d_f) = \begin{cases} 
\begin{align*}
p_dE(\tilde{e}) R^d_f &\geq 0 & \text{if } R^d_f \leq y_d/e, \\
((1 - \pi)p_d/e) R^d_f &\geq 0 & \text{if } y_d/e < R^d_f \leq y_de.
\end{align*}
\end{cases}$$

For our purposes the study of the behavior of $L_d(R^d_f)$ will be of major interest. It actually resembles the similar function of section 2. The right side of the diagram in Fig. 3 represents its general shape. The function is piecewise linear and discontinuous at $R^d_f = y_d/e$, where it has a local maximum equal to the left side limit:

$$L_d((y_d/e)--) = p_d y_d E(\tilde{e})/e > L_d((y_d/e)+) = (1 - \pi)p_d y_d/e^2$$

since $E(\tilde{e})e > 1 - \pi$. The maximum is global as well as local if

$$L_d((y_d/e)--) = p_d y_d E(\tilde{e})/e > L_d(\text{ye}) = (1 - \pi)p_d y_d \Leftrightarrow E(\tilde{e})/e > (1 - \pi).$$

In other words, this possibility (as shown in Fig. 3) is more likely the higher is the probability of depreciation of local currency $\pi$.

We can define in an obvious fashion $V_d(R^d_d) = \max\{p_d(y_d - R^d_d), 0\}$ (see the left side of the diagram in Fig. 3). The currency funding does not matter if $V_d(R^d_d) = V_d(R^d_f)$. This gives rise to the interest rates parities between matching and non matching rates are
\[ R_d^f = E(\bar{\varepsilon})R_d^f \text{ if } R_d^f \leq y_d/e, \]
\[ R_d^f = ((1 - \pi)/e)R_d^f \text{ if } y_d/e < R_d^f \leq y_de. \]

The analysis of the mismatched funding of a foreign project with domestic currency follows similar lines and the functions have similar properties. The expected net value of the project conditional on the \( R_d^f \) is now given by

\[ V_f(R_d^f) = \begin{cases} 
  p_f(y_f - R_d^f E(1/\bar{\varepsilon})) \geq 0 \text{ if } R_d^f \leq y_f/e, \\
  \pi p_f(y_f - R_d^f/e) \geq 0 \text{ if } y_f/e < R_d^f \leq y_f e.
\end{cases} \]

The expected return of the lender from a mismatched loan to sector

\[ L_f(R_d^f) = \begin{cases} 
  p_f R_d^f E(1/\bar{\varepsilon}) \text{ if } R_d^f \leq y_f/e, \\
  \pi p_f R_d^f/e \text{ if } y_f/e < R_d^f \leq y_f e.
\end{cases} \]

The interest rates parities between matching and non matching rates are

\[ R_f^f = E(1/\bar{\varepsilon})R_d^f \text{ if } R_d^f \leq y_d/e, \]
\[ R_f^f = (\pi/e)R_d^f \text{ if } y_d/e < R_d^f \leq y_de. \]

Finally, the maximum expected lending return in a perfectly matched loan is higher than the return of mismatched funding: \( p_d y_d \geq L_d(R_d^f) \) and \( p_f y_f \geq L_f(R_d^f) \).

In general, \( E(1/\bar{\varepsilon}) \geq 1/E(\bar{\varepsilon}) \), but sometimes we will take the approximation \( E(1/\bar{\varepsilon}) \approx 1/E(\bar{\varepsilon}) \).

What are the incentives to match the funding currency with the cashflow currency? It is clear that at low levels of interest rates, i.e. \( R_f^f \leq y_d/e \) and \( R_d^f \leq y_f/e \), the currency of the funding is irrelevant as long as the interest rates free of default risk are aligned according to the well known uncovered parities.

If interest rates are higher, i.e. \( R_f^f \geq y_d/e \) and \( R_d^f \geq y_f/e \), to gamble on the depreciation of the mismatched currency loan increases the value of the project:

\[ V_d(R_f^f) \geq p_d(y_d - R_d^f) \text{ with } R_d^f = E(\bar{\varepsilon})R_f^f, \]
\[ V_f(R_d^f) \geq p_f(y_f - R_d^f) \text{ with } R_d^f = E(1/\bar{\varepsilon})R_f^f. \]

This can be thought of as a sort of moral hazard. This is also mirrored in the discontinuous fall of the expected lender’s return from mismatched loans at interest rates just right above \( y_d/e \) and \( y_f/e \):
This basically means that the interest rates for perfectly matched loans $R_i^d$ relative to the mismatched ones $R_j^f$ become more expensive for the higher end of all rates. Competition would lead to different parities lowering $R_i^d$ relative to $R_j^f$: $R_i^d = ((1 - \pi)/e)R_i^f$ and $R_j^f = (\pi/e)R_j^f$.

A summary of much of this discussion is the following proposition:

**Proposition 1** The higher $e$ in a scenario of currency depreciation (the same for both currencies in this model), the more severe is the moral hazard of mismatched currency borrowing. As the probability of depreciation increases, the moral hazard will become more likely in the sector whose cashflows are in the currency which will appreciate accordingly.

We will now explore the effects of different assumptions on lenders’ attitudes toward risk on the currency matching outcomes: risk neutrality and risk aversion. In all cases we will assume that there are two large populations of perfectly competitive lenders: each has access to an outside option denominated in one currency. So, speaking loosely, there will be ”foreign” and ”domestic” lenders along with ”foreign” and ”domestic” projects, albeit all inhabit the same economy.

### 2.1 Risk neutrality

Foreign lenders have an outside option whose expected return in the same currency is $\varrho_f$ with $0 \leq \varrho_f \leq p_f y_f$, while domestic ones $\varrho_d$ with $0 \leq \varrho_d \leq p_d y_d$. Given the ”knife edge” nature of risk neutral decisions, we will assume that the outside options are ”arbitraged” in a risk free way: $\varrho_f \bar{E}(\tilde{e}) = \varrho_d$ or $\varrho_f = \varrho_d \bar{E}(1/\tilde{e})$. In this way, both types of lenders can operate in the market simultaneously. All lenders can invest in either project and agree to be repaid in any currency.

The credit market equilibria is represented in Fig. 3 where we can see the lenders’expected returns from funding the domestic project in foreign currency (on the right) or in local currency (on the left). A similar representation applies to the foreign project. We classify our equilibria in two different types

**a) There equilibria where at least one project is funded with a mismatched currency loan.**

This is the case for ”intermediate values” of the outside options such as $\varrho_2$ and $\varrho_3$ in Fig. 3, with the corresponding equilibrium points U2 and U3. For that we need that at least one of the equations

\[
L_d((y_d/e)−) > L_d((y_d/e)+) \quad \text{and} \quad L_f((y_f/e)−) > L_f((y_f/e)+).
\]
\[ \varrho_d = L_d(R_d^f) \text{ for } y_d/e < R_d^f \leq y_d e \text{ and } \varrho_f = L_f(R_f^d) \text{ for } y_f/e < R_f^d \leq y_f e \]

have a solution. An equivalent condition is

\[
(1 - \pi) p_d y_d/e^2 < \varrho_d \leq (1 - \pi) p_d y_d \text{ or }
\]

\[
\pi p_f y_f/e^2 < \varrho_f \leq \pi p_f y_f.
\]

In general, the larger the depreciation \( e \), the more likely both inequalities will be satisfied. Also, the likelihood of meeting both inequalities is greater for mid range values of \( \pi \). On the other hand if the depreciation probability becomes extreme, \( \pi \rightarrow 0 \) or \( \pi \rightarrow 1 \), only one type of project may receive mismatched funding: the one whose cashflows will appreciate.

Suppose we have an equilibrium rate \( R_d^f \), i.e. \( \varrho_d = ((1 - \pi) p_d/e) R_d^f \). There will be always another equilibrium in which either the project is funded with a matched loan like in points M2 and M3 in Fig. 3 (\( \varrho_d = p_d R_d^d \) if \( \varrho_d > p_d y_d E(\bar{e})/e \) or the loan currency does not matter like in MU3 (\( \varrho_d = p_d R_d^d = p_d R_f^d E(\bar{e}) \) if \( \varrho_d \leq p_d y_d E(\bar{e})/e \)). If the probability of foreign currency appreciation \( \pi \rightarrow 1 \), then \( R_d^f \rightarrow y_d e \) and foreign currency credit will be not available for domestic projects. The reason is simple: foreign lenders only can expect repayment from mismatched loans when the outcome is a depreciation of their currency. Thus we have for domestic lenders the two equilibria returns are given by

\[
\varrho_d = p_d R_d^d = p_d R_f^d E(\bar{e}) = ((1 - \pi) p_d/e) R_d^f.
\]

But these lenders can also invest in foreign projects competing with foreign creditors. The latter one may offer a mismatched currency loan with at an equilibrium rate of \( R_d^f \) such that \( \varrho_f = (\pi p_d/e) R_d^f \) if \( \pi p_f y_f/e^2 < \varrho_f \leq \pi p_f y_f \). In this case there is also another equilibrium in which either foreign projects borrowing in the same and a rate \( R_f^d \) such that \( \varrho_f = p_f R_f^d \) if \( \varrho_f > p_f y_f E(1/\bar{e})/e \) or the currency is immaterial for low values of \( \varrho_f \) (\( \varrho_f \leq p_f y_f E(1/\bar{e})/e \)). So the two equilibria case is

\[
\varrho_f = p_f R_f^d = p_f R_f^d E(1/\bar{e}) = (\pi p_d/e) R_d^f.
\]

It may be the case an equilibrium with a mismatched loan rate \( R_f^d \) does not exist; therefore, we are left with the other exclusive equilibria \( \varrho_f = p_f R_f^d = p_f R_d^d E(1/\bar{e}) \).
b) Every project is funded with either only matched loans or with loans in any currency when interest rates are aligned according default free risk uncovered parities, $R^d_d = E(\tilde{e})R^d_f$ or $R^f_f = E(1/\tilde{e})R^d_f$.

Looking at Fig. 3, we see that this is the case if the outside options are in the lower ($\varrho_4$) or in the upper end of the range ($\varrho_1$). If the two following inequalities

$$\varrho_d > L_d(R^d_f) \text{ for } R^d_f \leq y_{de},$$

$$\varrho_f > L_f(R^f_d) \text{ for } R^f_f \leq y_{fe},$$

hold true, then corresponding project $d$ and $f$, are only funded with matched loans (point M1). Equivalent inequalities are

$$\varrho_d > p_d y_d \max \{1 - \pi, E(\tilde{e})/e\},$$

$$\varrho_f > p_f y_f \max \{\pi, E(1/\tilde{e})/e\}.$$ 

While in the previous situation the opportunity costs $\varrho_d$ and/or $\varrho_f$ are ”too high”, if these ones are ”low”, the currency of the loan does not make any difference as to the probability of bankruptcy of the project. More specifically, if the two inequalities

$$\varrho_d \leq (1 - \pi)p_d y_d/e^2,$$

$$\varrho_f \leq \pi p_f y_f/e^2$$

are satisfied, the currency of the loan is irrelevant provided that the interest rates are aligned according to the parities $R^d_d = E(\tilde{e})R^d_f$ or $R^f_f = E(1/\tilde{e})R^d_f$, as shown in points M4 and MU4.

**Summing up:**

If the opportunity costs $\varrho_d$ and $\varrho_f$ are either in the lower or upper end of the feasible range, there will be a unique equilibrium with perfect currency matching, or if the loan is mismatched, the exchange risk will not alter the default probability of the project.

Alternatively, if $\varrho_d$ and $\varrho_f$ are in the ”mid” range, there will be two equilibria. One has the same properties as the equilibrium for extreme values of opportunities costs already described, and the other is a mismatched lending.

Finally, we have a mixed case with one opportunity cost in the mid range and the other in either extreme.
2.2 Risk aversion

We now discuss the effects of lenders’ risk aversion. Let \( u(\cdot) \) be the strictly concave and continuous utility function for all lenders population. Let \( U_d \) and \( U_f \) be the expected utility values of the outside options for the holders of domestic and foreign currency respectively. Defining the outside options as random variables \( \tilde{\varrho}_d \) and \( \tilde{\varrho}_f \) independent of \( \tilde{e} \), we have

\[
U_d = E[u(\tilde{\varrho}_d)] \quad \text{and} \quad U_f = E[u(\tilde{\varrho}_f)].
\]

The opportunities are ”arbitraged”, that is, each lender type is indifferent between his own currency opportunity and the other one:

\[
U_d = E[u(\tilde{\varrho}_d)] = \pi E[u(\tilde{\varrho}_f/e)] + (1 - \pi) E[u(\tilde{\varrho}_d/e)],
\]

\[
U_f = E[u(\tilde{\varrho}_f)] = \pi E[u(\tilde{\varrho}_d/e)] + (1 - \pi) E[u(\tilde{\varrho}_f/e)].
\]

Arguing in a similar fashion to the risk neutrality case, we notice that any lender can agree with the borrower to be repaid in either currency and invest in either project. Just for the sake of concreteness always take \( p_d > p_f \).

The expected utility of a domestic lender from funding foreign project in domestic currency is (shown on the right side of Fig. 4):

\[
RAL_f(R^f_d) = \begin{cases} 
p_f u(R^f_d) & \text{if } R^f_d \leq y_f/e, \\
\pi p_f u(R^f_d) & \text{if } y_f/e < R^f_d \leq y_f e.
\end{cases}
\]

It is just a version of \( L_f(R^f_d) \) for the risk neutral case. The maximum of \( RAL_f(R^f_d) \) is either at the discontinuity point \( y_f/e \) or at the upper corner \( y_f e \):

\[
\max RAL_f(R^f_d) = \max\{p_f u(y_f/e), \pi p_f u(y_f e)\} = p_f \max\{u(y_f/e), \pi u(y_f e)\}.
\]

If the loans are instead channeled to the domestic project, we have (see again the left side of Fig. 4):

\[
RAL_d(R^d_d) = p_d u(R^d_d) \quad \text{with } R^d_d \leq y_d.
\]

Given our assumptions, \( \max RAL_d(R^d_d) = p_d u(y_d) \geq RAL_f(R^f_d) \).

Similarly, foreign lenders’ expected utility from mismatched loans (right side of Fig. 5) is

\[
RAL_d(R^d_f) = \begin{cases} 
p_d u(R^d_f) & \text{if } R^d_f \leq y_d/e, \\
(1 - \pi) p_d u(R^d_f) & \text{if } y_d/e < R^d_f \leq y_d e.
\end{cases}
\]
and the properties are similar to those ones of $RAL_d(R^d_f)$. For matching loans, we can define $RAL_f(R^f_d) = p_f u(R^f_d)$ (left side of Fig. 5) To reduce the number of parameter combinations to discuss we assume that $p_f < (1 - \pi)p_d$, that is, the depreciation probability $\pi$ is not "too large".

We discuss the outcomes of the model according to the value of the outside options. The broad classification of equilibrium is the same as in the risk neutral case, but we will reverse the presentation of the cases.

**a) Every project is funded with matched loans or with loans in any currency when interest rates are aligned according default free risk uncovered parities.**

This happens for extreme values of the outside options (out of the vertical interval $[A,B]$ in Figs. 4 and 5). Let us start by the lower end:

$$\pi p_f u(y_f/e) \geq U_d \text{ and } p_f u(y_f) \geq U_f.$$ 

Domestic lenders are willing to make loans in all currencies to all sectors at rates which must satisfy:

$$U_d = p_d u(R^d_d) = p_d E(u(R^d_d)) = p_f u(R^f_d) = p_f E(u(R^f_d)).$$

An elementary application of the Jensen’s inequality leads to

$$R^d_d \leq R^d_f E(\tilde{e}) \text{ or } R^d_d \leq R^d_f ((1 - \pi)/e),$$

$$R^f_d \leq R^f_f E(\tilde{e}).$$

These lenders are only willing to make mismatched loans if they can charge premia for exchange risk (plus for an eventual increase in credit risk). Risk neutral borrowers will reject these loans. Thus in equilibrium lenders make loans in their currency type. Then

$$U_d = p_d u(R^d_d) = p_f u(R^f_d).$$

Similar considerations lead to foreign lenders offering contracts in their currency at rates satisfying

$$U_f = p_d u(R^d_d) = p_f u(R^f_f).$$

Interest rates reflect default probabilities ($p_i > p_j \iff R^i_d > R^j_d$) and a premium for credit risk given by the shape of $u(\cdot)$. Furthermore, low opportunity costs $U_i$ result in interest rates at such a low levels that mismatched currency funding do not increase project default probabilities.
It is clear that borrowers have a choice as to project finance: match vs. mismatch the currency loan with the project currency cashflows. A careful look at the ratios

\[
\frac{U_d}{U_f} = \frac{u(R_d^d)}{u(R_f^d)} = \frac{u(R_d^f)}{u(R_f^f)}
\]

provides the key of borrowers’ behavior. Let us consider first the simpler version

\[
\frac{U_d}{U_f} = \frac{u(R_d)}{u(R_f)}
\]

The LHS is the relative expected values of the outside investment options of both currencies. Given the expected future exchange rate \(E(\tilde{e})\), the higher \((U_d/U_f)\), the higher lenders’ ask quote of domestic funding relative to foreign one \(R_d/R_f\). Conversely, the higher \(E(\tilde{e})\) with respect to a given \((U_d/U_f)\), the more costly become foreign currency loans.

Suppose that, by chance, we obtain

\[
\frac{U_d}{U_f} = \frac{u(R_d)}{u(R_f)}
\]

and

\[
\frac{R_d}{R_f} = E(\tilde{e}),
\]

then risk neutral borrowers would be indifferent between taking loans in one or the other currency. Let us replace \(R_d\) with \(R_fE(\tilde{e})\) and compute the ratio

\[
\frac{u(R_fE(\tilde{e}))}{u(R_f)}.
\]

If

\[
\frac{U_d}{U_f} > \frac{u(R_fE(\tilde{e}))}{u(R_f)},
\]

then it would be implicit \(R_d > R_fE(\tilde{e})\) and risk neutral borrowers take foreign currency loans. Alternatively, if

\[
\frac{U_d}{U_f} < \frac{u(R_fE(\tilde{e}))}{u(R_f)},
\]

then \(R_d < R_fE(\tilde{e})\) and projects would be funded in local currency. With this analysis we can elaborate the following taxonomy:
**Case 2** One currency dominates all loans if either

\[
\frac{U_d}{U_f} > \max \left\{ \frac{u(R_d^f E(\tilde{e}))}{u(R_d^f)}, \frac{u(R_d^f E(\tilde{e}))}{u(R_d^f)} \right\} \quad \text{or} \quad \frac{U_d}{U_f} < \min \left\{ \frac{u(R_d^f E(\tilde{e}))}{u(R_d^f)}, \frac{u(R_d^f E(\tilde{e}))}{u(R_d^f)} \right\}.
\]

The first inequality implies that all projects are funded in foreign currency and the second one in local currency. In either case one type of lenders is driven out of the market. We know that risk averse lenders always match, but in each instance only one type of borrowers do, and therefore, we can speak of ”perfect currency matching”.

**Case 3** Both types of lenders participate simultaneously if either

\[
\frac{u(R_d^f E(\tilde{e}))}{u(R_d^f)} > \frac{U_d}{U_f} > \frac{u(R_d^f E(\tilde{e}))}{u(R_d^f)} \quad \text{or} \quad \frac{u(R_d^f E(\tilde{e}))}{u(R_d^f)} < \frac{U_d}{U_f} < \frac{u(R_d^f E(\tilde{e}))}{u(R_d^f)}.
\]

The first line of inequalities implies that domestic projects are financed with loans in local currency, and foreign projects in foreign currency, and so achieving ”perfect currency matching”. Instead, the second line of inequalities implies that all projects are funded with mismatched currency loans. Anyway, all lenders operate in the market, but they specialize by type of project and currency.

Previous analysis suggest a different characterization of ”arbitraged” outside options.

**Definition 4** Let \( \alpha(x) = u(E(\tilde{e})x)/u(x) \quad \forall x \geq 0 \). If \( \exists \tilde{x} \geq 0 \text{ such that } \alpha(\tilde{x}) = U_d/U_f \), then it is said the outside options \( d \) and \( f \) are arbitraged.

**Corollary 5** If outside options \( d \) and \( f \) are arbitraged, then either

\[
\max\{U_d/U_f, E(\tilde{e})\} \leq 1 \text{ or } \min\{U_d/U_f, E(\tilde{e})\} > 1.
\]

The idea is that, if both options are not ”arbitraged”, only one type of currency loan will be around.

The following is a version of the case of low opportunity costs:
\[ \pi p_f u(\frac{y_f}{e}) \geq U_d, \]
\[ (1 - \pi) p_d u(\frac{y_d}{e}) \geq U_f > p_f u(y_f). \]

The point here is that foreign lenders cannot lend to foreign projects. Their only possibility is to make mismatched loans to the domestic sector \( U_f = p_d u(R_d^f) \). But he is competing with domestic lenders offering contracts at a rate \( R_d^d \). An analysis of the ratios

\[ \frac{U_d}{U_f} = \frac{u(R_d^d)}{u(R_f^d)} \text{ and } \frac{u(R_d^d E(\bar{e}))}{u(R_f^d)} \]

provides the answer as to whether both types of lenders participate or not in the market.

Let us now turn to the upper end of the opportunity cost range. The "innocuous assumption" \( p_d > p_f \) leads to the following definition of this region of the parameter space:

\[ p_d \max\{u(\frac{y_f}{e}), \pi u(\frac{y_f}{e})\} < U_d \]
\[ \max\{(1 - \pi) p_d u(\frac{y_d}{e}), p_f u(y_f)\} < U_f \leq p_d u(\frac{y_d}{e}). \]

In this region domestic lenders can make loans to domestic borrowers at a rate \( R_d^d \) such that \( U_d = p_d u(R_d^d) \), and foreign lenders can offer contracts to the same borrowers at a rate \( R_f^d \) such that \( U_f = p_d u(R_f^d) \). As in previous instances, the analysis of the ratios

\[ \frac{U_d}{U_f} = \frac{u(R_d^d)}{u(R_f^d)} \text{ and } \frac{u(R_d^d E(\bar{e}))}{u(R_f^d)} \]

will dictate what type of lenders will prevail. Shifting the bounds of the inequalities in the following fashion

\[ \pi u(\frac{y_f}{e}) < U_d \leq p_d u(\frac{y_f}{e}) \]
\[ (1 - \pi) p_d u(\frac{y_d}{e}) < U_f \leq p_f u(y_f), \]

we can have again the possibility that both types of lenders make loan offers to all projects:
\[ U_d = p_d u(R_{d}^{d}) = p_f u(R_{f}^{d}) \quad \text{and} \quad U_f = p_d u(R_{d}^{f}) = p_f u(R_{f}^{f}). \]

Notice how a difference in the probabilities of project success leads to the worse one to be driven out of the market for extreme configurations of the parameter space.

b) At least one project is funded with a mismatched currency loan.

So far mismatched project funding has not change the probability of borrowers’ bankruptcy. If any of the opportunity costs are in the following mid ranges (intervals [A,B] in Figs. 4 and 5).

\[
\pi_p f(y_f/e) < U_d \leq \pi_p f(y_f e)
\]

\[
(1 - \pi)p_d u(y_d/e) < U_f \leq (1 - \pi)p_d u(y_f e),
\]

there will be equilibria in which the exchange risk of mismatched loans will increase the default probability of the project.

Suppose that both opportunity costs satisfy the inequality. There is an equilibrium with ”high” interest rates \( R_{d}^{d} \) and \( R_{f}^{d} \) at which borrowers choose riskier mismatched currency loans:

\[ U_d = \pi_p f u(R_{d}^{d}) \quad \text{and} \quad U_f = (1 - \pi)p_d u(R_{f}^{d}). \]

Notice that lenders do not compete: each type lends to a different sector.

In the equilibrium with ”low” interest rates the exchange risk of eventual mismatched loans is immaterial. In our parameter space domestic lenders will offer loans with rates

\[ U_d = p_d u(R_{d}^{d}) = p_f u(R_{d}^{f}) \]

and foreign lenders will offer \( R_{f}^{d} \) and possibly \( R_{f}^{f} \) (if \( U_f \leq p_f u(y_f) \)) satisfying similar conditions. Here there is competition between both types of lenders to fund the domestic project.

Of course, there could be mixed cases. In the following region \( U_d \) is in the upper end and \( U_f \) is in the mid range:

\[
p_d \max\{u(y_f/e), \pi u(y_f e)\} < U_d \leq p_d u(y_d)
\]

\[
(1 - \pi)p_d u(y_d/e) < U_f \leq (1 - \pi)p_d u(y_f e).
\]
Domestic lenders offer loans at a rate $R_d$, and foreign lenders offer contracts at low rates in one equilibrium ($R_f^d$ and possibly $R_f^f$) and one a high rate $R_f$. Here, there is competition and either domestic or foreign lenders could be driven out of the market completely or in only one equilibrium.

**Summing up:**

We see that the story does not change very much although details are messier. As expected, lenders do match the currency of their claims against borrowers with the currency of their endowment. However, there still remains equilibria with mismatched currency funding.

A brief comment on risk taking behavior is in order. This assumption almost always seems a bit awkward. More often than not, it is a metaphor of institutions such as deposit insurance in banking or just simply limited liability. But the only effect here is to make lenders to choose exchange risk by lending in the currency with which they are not endowed.

3 Investment and loan currency choices

Here we combine the investment project selection problem of section 1 with currency choice problem of the preceding section. We make a simple change in our setup: now we define a firm as an agent making a choice between the domestic and foreign project. However, lenders will not be able to control that choice. Thus, we will have loan offers which depend only on the currency but not on the project ($R_d$ and $R_f$). Lenders are risk neutral.

The central point is that, in a well defined sense, the project ranking does not depend on the loan currency. The following proposition is a particular statement of the idea:

**Proposition 6**

$$V_d(R_f) \geq \max\{E(\tilde{e})p_f(y_f - R_f), 0\} \forall R_f \leq y_d$$

if and only if

$$p_d(y_d - R_d) \geq E(\tilde{e})V_f(R_d) \forall R_d \leq y_d.$$

**Proof.** The "if" part is trivial. As long as the interest rates obey the default risk free uncovered parity $R_d = E(\tilde{e})R_f$, having a currency choice cannot be worse than perfect currency matching. Therefore, $V_d(R_f) \geq p_d(y_d - R_d)$ and $V_f(R_d) \geq p_f(y_f - R_f)$.

The "only if" part is only slightly less trivial. By hypothesis, $V_d(0) = p_d y_d \geq E(\tilde{e})p_f y_f$. Suppose now that for some $R_d$ we have $p_d(y_d - R_d) < E(\tilde{e})V_f(R_d)$. Since $V_f(R_d)$ is convex and $V_f(y_f e) = V_f(y_d) = 0$, then $p_d y_d < E(\tilde{e})V_f(0) = E(\tilde{e})p_f y_f$, which is a contradiction. ■

The proposition depicts a case in which one project dominates the other one regardless of the interest rates. The only moral hazard problem is the
incentive to choose a riskier mismatched funding if the interest rates are high enough.

If project dominance depends on interest rates, the following claim applies:

**Proposition 7** Suppose there exists a unique strictly positive domestic interest rate \( R_d \neq y_d \) such that \( p_d(y_d - \hat{R}_d) = E(\tilde{e})V_f(\hat{R}_d) \). Then \( \hat{R}_d < y_f/e \) and for all \( R_d < \hat{R}_d \) the foreign project will be chosen. The domestic project, the riskier one, will be chosen if \( R_d > \hat{R}_d \). Furthermore, there exists a foreign interest rate \( \hat{R}_f < y_d/e \) such that \( V_d(\hat{R}_f) = E(\tilde{e})p_f(y_f - \hat{R}_f) \) and \( \hat{R}_d = \hat{R}_f E(\tilde{e}) \). Project choices will be consistent with the ones determined by \( \hat{R}_d \): the domestic project will be selected for \( R_f > \hat{R}_f \) and will be funded in a mismatched fashion for \( R_f > y_f/e \).

**Proof.** We show first that \( \hat{R}_d < y_f/e \). It is more convenient to write \( E(\tilde{e})V_f(\hat{R}_d) = p_f(y_f E(\tilde{e}) - \hat{R}_d) \) for \( \hat{R}_d \leq y_f/e \). It must be the case that at \( R_d = 0 \)

\[
E(\tilde{e})p_f y_f > p_d y_d = p_d y_f e \implies E(\tilde{e})p_f > p_d e \implies p_f > p_d.
\]

As defined in section 2, the foreign project is riskier. At \( R_d = y_f/e \), we have \( p_d(y_d - y_f/e) > E(\tilde{e})V_f(y_f/e) \).

Together the convexity of \( V_f(\cdot) \) we conclude that \( \hat{R}_d < y_f/e \). By explicit calculation

\[
p_d(y_d - \hat{R}_d) = p_f(y_f E(\tilde{e}) - \hat{R}_d) \implies \hat{R}_d = \frac{E(\tilde{e})p_f y_f - p_d y_d}{p_f - p_d} = \frac{y_f E(\tilde{e})p_f - p_d e}{p_f - p_d}.
\]

Clearly, \( p_d(y_d - \hat{R}_d) < p_f(y_f E(\tilde{e}) - R_d) \) for \( R_d < \hat{R}_d \). That means the foreign project will be chosen for ”low” values of \( R_d \).

The mere computation of \( \hat{R}_f \) establishes its existence:

\[
V_d(\hat{R}_f) = p_d(y_d - \hat{R}_f E(\tilde{e})) = E(\tilde{e})p_f(y_f - \hat{R}_f) \implies
\]

\[
\hat{R}_f = \frac{E(\tilde{e})p_f y_f - p_d y_d}{E(\tilde{e})(p_f - p_d)} = \frac{y_f E(\tilde{e})p_f - p_d e}{E(\tilde{e})(p_f - p_d)}
\]

and then \( \hat{R}_d = \hat{R}_f E(\tilde{e}) \). \( \blacksquare \)
The dominance of either project depends on the free parameters $p_f$, $p_d$, and $\pi$. However, the reader can ask what feature of the environment drives the domestic project as riskier under the previous proposition. The answer lies in the assumption $y_d = y_f e$. The point here is that $e > E(\tilde{e})$ and then $y_d > y_f E(\tilde{e})$. As result we got a domestic project with a higher cashflow but with lower probability of success. Needless to say, nothing important is at stake in this model with the particular set of parameters chosen.

It is convenient to specify the loan repayment in domestic currency and as a function of the foreign interest rate $R_f$:

$$L(R_f) = \begin{cases} p_f R_f E(\tilde{e}) & \text{if } R_f \leq \tilde{R}_f, \\ p_d R_f E(\tilde{e}) & \text{if } \tilde{R}_f < R_f \leq y_d / e, \\ (1 - \pi)p_d (R_f / e) & \text{if } R_f > y_d / e. \end{cases}$$

Figs. 6A, 6B and 6C represent three generical cases. We classify the equilibria according to where the outside option $\varrho_d$ lies:

1. There is a unique equilibrium in which the foreign project gets selected and the loan currency does not matter if the outside option is low (point A in Fig. 6A):

$$\varrho_d \leq \min\{p_d E(\tilde{e}) \tilde{R}_f, (1 - \pi)p_d E(\tilde{e})(y_d / e)\},$$

or high (point B in the same Fig.):

$$p_f E(\tilde{e}) \tilde{R}_f = \max\{p_f E(\tilde{e}) \tilde{R}_f, p_d E(\tilde{e})(y_d / e), (1 - \pi)p_d y_d\} \geq \varrho_d > \max\{p_d E(\tilde{e})(y_d / e), (1 - \pi)p_d y_d\}.$$  

2. There is a unique equilibrium in which the domestic project is chosen if the outside option is in the mid range

$$\max\{p_d E(\tilde{e})(y_d / e), (1 - \pi)p_d y_d\} \geq \varrho_d > \min\{p_d E(\tilde{e})(y_d / e), (1 - \pi)p_d y_d\}.$$  

If $p_d E(\tilde{e})(y_d / e) = \max\{p_d E(\tilde{e})(y_d / e), (1 - \pi)p_d y_d\}$, like in point A of Fig. 6B, the loan currency is irrelevant. Otherwise, the borrower chooses a mismatched loan (point A in Fig. 6C)

3. A more likely outcome for intermediate values of $\varrho$ is, whose maximum number could be three if

$$\min\{p_f E(\tilde{e}) \tilde{R}_f, p_d E(\tilde{e})(y_d / e), (1 - \pi) p_d y_d\} \\ \geq \varrho_d > \max\{p_d E(\tilde{e})(y_d / e), (1 - \pi)p_d y_d\}.$$  

An example is points a, b, and c in Fig. 6A. As a final remark, if the probability of domestic currency depreciation $\pi \rightarrow 1$, then both types of moral hazard merge into one ($\tilde{R}_f \rightarrow y_d / e$).
References


GRAPHICAL APPENDIX
Fig. 1: Equity value of the investment project choice in a one currency economy as a function of $R$. 

The diagram illustrates the relationship between the value function $V(R)$ and the parameters $p_y y_g$ and $p_b y_b$. The point $A$ represents the initial value, and the point $C$ represents the value at the end of the period. The point $B$ indicates the value at a certain intermediate point $R_0$. The horizontal axis represents $R$, and the vertical axis represents $V(R)$. The dotted lines represent the budget constraints for different values of $R$. 

The diagram shows how the equity value changes as the risk parameter $R$ varies.
Fig. 2: Credit market equilibria in a one currency economy
Fig. 3: Credit market equilibria in a dual currency economy: Domestic project funding

\[ L_d(R_f^d) \]

\[ L_d(R_f^d) \]

\[ y_d \]

\[ y_{de} \]

\[ \rho_1 \]

\[ \rho_2 \]

\[ \rho_3 \]

\[ \rho_4 \]

\[ R_{d}^d \]

\[ R_{q}^d \]
Fig. 4: Domestic lenders' expected utility from funding domestic and foreign projects
Fig. 5: Foreign lenders' expected utility from funding domestic and foreign projects
Fig. 6A: Lenders' expected returns from funding the best firm's project choice

\[ L(R_f) \]

\[ R_{f0} \]

\[ y_f \]

\[ y_d \]

\[ \rho_1 \]

\[ \rho_2 \]

\[ \rho_3 \]
Fig. 6B: Lenders' expected returns from funding the best firm's project choice
Fig. 6C: Lenders' expected returns from funding the best firm's project choice